PRINCIPLES

OF

ARCHITECTURAL PERSPECTIVE.

LAWRENCE.

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PRINCIPLES

OF

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ARCHITECTURAL PERSPECTIVE.

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ST. JOSEPH'S UNIVERSITY STX
NA2710.L42
Principles of architectural perspective.
RELEASE
3 9353 00091 9835

FOURTH EDITION.

BOSTON:

W. H. LAWRENCE
491 Boylston Street
1915

NA 2710
L 42

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INTRODUCTION.

This little book has been prepared for use in scientific and technical schools, where it is desired to give a short but comprehensive course in perspective. It has been the aim of the author to make the text as concise as possible, consistent with clear and complete explanation. The primary object of the book is to reduce the amount of note-taking incidental to a lecture course, and to give the student a more tangible and satisfactory reference than his own always hurriedly and sometimes poorly written notes. The explanations and examples here given should be supplemented by illustrative problems of a practical nature, devised by the instructor for the solution of the student.

In the opinion of the writer, the text-book is not the proper place for these practical problems. They should be varied to suit the needs of each class. Consequently, no attempt has been made to define a fixed course for the drawing room. Whatever special method the instructor may favor, an intelligent consideration of the practical side of the subject must be based upon a knowledge of the underlying principles. The exposition of these principles is all that has been undertaken here. It is hoped that the book will cover a broader field and lead to a more intelligent study of perspective than if it had been planned to meet the demands of the class of students who want simply to know *how* to do, and care not *why*.

It may be said that there is essentially but one fundamental phenomenon of perspective; viz., the apparent diminution in the size of an object as it recedes from the eye of the observer. The whole theory of the subject is dependent upon this illusion and may be developed from it step by step.

As perspective projection is simply one branch of descriptive geometry, the writer believes that the only logical way to approach the subject is from a strictly geometrical point of view. Regarded in this manner, the underlying principles are few in number and

extremely simple. In fact, all the operations in making a perspective drawing reduce themselves to the rudimentary problem in descriptive geometry, of finding the intersection of a right line with a plane. Understanding this one problem, the student should have no difficulty in following the explanations given in these pages.

The first chapter of the book is devoted to definitions and to the theory of perspective in general.

In the second chapter are given ten elementary problems covering all of the descriptive geometry that is needed to completely solve a problem in perspective. All of these elementary problems will be found to depend directly upon the problem of finding the intersection of a right line with a plane. The student is advised to make himself thoroughly familiar with the contents of this chapter before attempting the solution of the practical problem which is given in Chapter III.

Chapter IV. follows with two general problems. In each case the complete *vanishing-point diagram* (§ 69) has been found and drawn in red. A third problem in Chapter IV. illustrates the so-called "Parallel" or "One Point" perspective.

Much stress has been laid on general methods, and the student should completely master them. He will then be in a condition to study and understand the methods of direct division, the relation between lines in perspective, the direct measurement of lines, and the method of perspective plan, which are discussed in Chapters V., VI., VII., VIII., and IX.

Most of the short cuts in making a perspective drawing are based upon the principles given in these chapters. Without a knowledge of the general theory of perspective, these short-cut methods are usually remembered as rules of thumb, and are thus not infrequently employed where their application is not strictly correct. With the understanding that should come from the study of the previous chapters, and the hints given in Chapters V., VI., VII., and VIII., the student, with a little ingenuity, will be able to devise his own short-cut methods, to appreciate their philosophy, and to vary their applications to meet the requirements of the infinite number of problems that will arise in practice.

Chapter X. treats of curves.

Chapter XI. shows how to find shadows in perspective.

Chapter XII. is a discussion on apparent distortion.

There seems to exist in the minds of some beginners in the study of perspective the idea that the drawing of an object made in accordance with geometrical rules differs in many essentials from the object as seen in nature. Such an idea is entirely erroneous, however. The only difference between a view in nature and its correctly constructed perspective projection is that the view in nature may be looked at *from any point*, while its perspective representation shows the view as seen from some *one particular point*.

Before making a perspective drawing, the point of sight, or station point, as it is called, must be decided upon, and the resulting perspective projection will represent the object as seen from this point, and from this point only. The view in nature will present a different appearance to the observer for every new position which he takes, but the drawing which has been made upon a sheet of paper is fixed and evidently *cannot represent the view as seen from more than one point.*

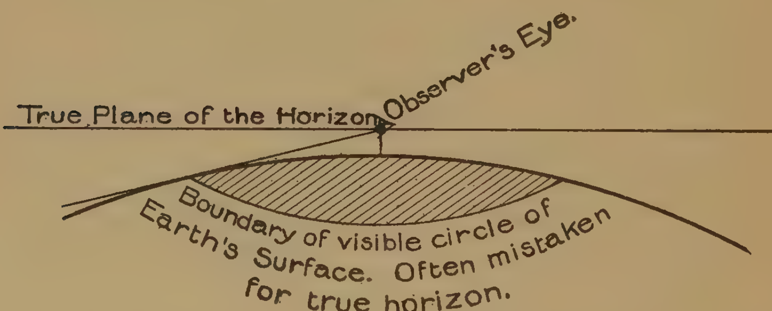
This fact should be borne in mind when looking at a perspective drawing. If the observer will take pains to place his eye in exactly the position which was assumed for it when making the drawing, all lines and points in the perspective representation will appear to him to bear exactly the same relation to one another that exists in the object in nature *when viewed from a corresponding position.*

In most of the plates in this book, the station point has been assumed much nearer the plane of projection than is generally advisable. This has been done in order to show the whole construction upon the sheet. The resulting perspective in some cases appears much distorted owing to the fact that the observer's eye is at some distance from the station point while looking at the drawing. But even in the most extreme cases, if the observer will place his eye in the position in which it was assumed to be when making the drawing, all distortion will disappear. For illustration consider Fig. 27. The station point in this figure has been chosen very near the plane of projection, and as a result, the perspective of the object appears much distorted. But for the sake of experiment, let the student cut a small round hole from a piece of cardboard, place the hole directly in front of SP^V and about one and a half inches from it, and look at the drawing through the hole. His eye is now exactly at the assumed position of the station point and he will notice that all distortion has disappeared. Other illustrations of this same phenomenon will be found in the figures illustrating distortion in perspective.

It will thus be seen that the disagreeable effects so often noticed in a perspective drawing are due, not to faults in the science of perspective, but rather to the fault of the artist in unwisely choosing the position of the station point.

It often puzzles a beginner, especially if he has done some sketching out of doors, to know why a straight line is used for a horizon, instead of the "curved horizon line of nature." As a fact, the *true horizon line of nature is a perfectly straight horizontal line*, or what is exactly the same thing, it is a horizontal circle of infinite radius, the centre of which is the observer's eye.

The circular line (where the sea and sky seem to meet) which is so often mistaken for the true horizon, has in reality no right whatever to the name. It is not the horizon, but the finite circle which forms the extreme boundary of the globe's surface, that is visible to the observer. It is projected upon the plane of projection or upon the retina of the eye just as any other finite circle in nature.



The true horizon is the *infinite boundary* of a horizontal plane supposed to pass through the observer's eye and extend indefinitely in all directions. This boundary will evidently appear to the observer as a perfectly straight line, and will be so projected upon the plane of projection, while the finite circle, which bounds the portion of the globe's surface seen by the observer will be projected as part of an hyperbola (§ 179). In theory these two lines should never be confused. But in practice they are so nearly coincident that no distinction is usually made between them.

If constructed mathematically, the hyperbolic line will have so

slight a curvature that even at the extreme edges of a large drawing it will depart from the horizontal but an inappreciable amount, and will coincide so very nearly with the perspective of the true horizon that, for all practical purposes, the two may be considered as one.

Understanding this, and realizing that the limitations of a drawing upon a plane surface make it necessary to place the eye at a certain fixed point while viewing the drawing, it may safely be stated that an accurately constructed perspective projection is an exact representation of the corresponding view as seen in nature.

PRINCIPLES OF ARCHITECTURAL PERSPECTIVE.

CHAPTER I.

DEFINITIONS AND GENERAL THEORY.

1. — Perspective is the science of representing upon a plane surface, objects at various distances, as they appear to the eye from a given point of view.

2. — Everyone is probably familiar with some of the more evident phenomena of perspective. Perhaps the most striking, and certainly the most important of these, is the apparent diminution in the size of an object as it recedes from the eye. A railroad train moving over a long straight track furnishes an excellent example of this. As the train becomes more and more distant, its dimensions apparently become smaller and smaller, the details grow more and more indistinct, and finally the whole train appears like a black line crawling over the ground. It will be noticed, also, that the speed of the train seems to diminish as it moves away, for the space over which it travels in a given time seems less and less as it is taken farther and farther from the eye.

3. — In the same way, if several objects having the same dimensions are situated at different distances from the eye, the nearest one appears to be the largest, and the others appear to be smaller and smaller as they are farther and farther away.

Take, for instance, a long straight row of street lamps. As one looks along the row each succeeding lamp post is apparently shorter and smaller than the one before. The reason for this can readily be explained. In estimating the size of any object, one most naturally compares it with some other object as a standard or unit. Now suppose the observer compares the lamp posts one with another, the result will be something as follows: —

(Fig. 1.) Suppose he first looks at the top of No. 1, along the line *ba*. The top of No. 2 is invisible. It is apparently below the top of No. 1, and in order to see it he has to lower his eye until he is looking in the direction ba_1. He now sees the top of No. 2, but the top of No. 1 seems some distance above, and he naturally concludes that No. 2 is shorter than No. 1. As the observer looks at the top of No. 2, No. 3 is still invisible, and in order to see it he has to lower his eye still farther. Comparing the bottoms of the posts he finds the same apparent shrinkage in size as the distance of the post from his eye increases.

4. — abc, a_1bc_1, and a_2bc_2, are called the *visual angles* which are subtended by the posts. It will be seen that as the distance between the observer's eye and the object increases, the visual angle diminishes. *When this distance becomes infinite, the visual angle becomes zero and the object appears as a single point.*

5. — Parallel lines, as they recede from the eye, appear to converge, the distance between them seeming less and less as it is taken farther and farther away. At *infinity* this distance becomes *zero* and the lines appear to meet in a *single point.* This point is called the *vanishing point* of the lines.

6. — If an object is carefully studied it will be seen that its lines may be separated into groups according to their different directions, all the lines having the same direction forming one group and apparently converging to a common vanishing point. Each group of parallel lines is called a *system* and each line an *element* of the system. For example, in Fig. 2 A, A_1, A_2, and A_3 belong to one group or system; B, B_1, B_2, and B_3 to another; and C, C_1, C_2, and C_3 to a third.

7. — If the lines of a system extend indefinitely in both directions, they will appear to converge as one looks in either direction and seem to meet in two points at infinite distances from the eye, one being at each extremity of the system. Thus every system of lines will have two vanishing points 180° apart. As the principles of perspective are based upon the assumption that the observer remains perfectly stationary while viewing an object, we generally have to consider but one of these vanishing points, viz.: the one which lies in front of the observer.

8. — As all lines which belong to the same system must apparently meet at the vanishing point of that system, it follows that *if we look*

directly along any line of a system, we shall be looking directly at the vanishing point of that system; that is to say, the line along which we are looking will be seen *endwise* and will appear as a *single point exactly covering and coinciding with the vanishing point of the system to which it belongs.* Thus, to find the vanishing point of any system, imagine one of its elements to enter the observer's eye. This element will appear to him as a single point exactly covering the required vanishing point.

9. — All planes that are parallel to one another are said to belong to the same system. They appear to converge as they recede from the eye, and at infinity to meet in a *single straight line* called the *vanishing trace* of the system.

10. — If the eye is placed so as to look directly along one of the planes, that plane will be seen as a straight line exactly covering and coinciding with the vanishing trace of the system to which it belongs. If the observer is supposed to turn slowly around, still looking along the plane, its vanishing trace will always appear as a straight line directly in front of his eye. Thus the vanishing trace of any system of planes is really the circumference of a circle, the radius of which is infinity, its centre being the observer's eye.[1]

11. — It will thus be seen that all horizontal planes must vanish in a horizontal line directly in front and on a level with the observer's eye. This line is called the *horizon.* The horizontal plane which passes through the observer's eye is called the *plane of the horizon.*

12. — *It is evident that any line which lies in a plane must have its vanishing point in the vanishing trace of the plane.* Thus the horizon will contain the vanishing points of all *horizontal* lines.

13. — Conversely, *the vanishing trace of any plane must pass through the vanishing points of all lines that lie in it.* Thus the vanishing points of any two lines lying in a plane will determine the vanishing trace of the system to which the plane belongs.

14. — *As the intersection of two planes is a line lying in both, its vanishing point must lie at the intersection of the vanishing traces of the two planes.*

NOTE [1]. — A straight line may always be considered the circumference of a circle of infinite radius.

15. — An object becomes visible by means of rays of light which are reflected from its surface and enter the observer's eye. These rays are called *visual rays*. They form a cone or pyramid which has the object for its base and the observer's eye for its apex. (Fig. 3.) If any plane (M) is placed so as to intersect this cone or pyramid, the intersection will be seen as the perspective (P) of the object (O) upon the plane (M).[1] The plane (M) which receives this perspective projection is called the *picture plane*. Every point in the perspective will appear to the observer to exactly cover a corresponding point in the real object, and will thus present the same appearance to him as the real object in space.

16. — For every new position that the observer takes, he will see a new perspective projection of the object, his eye always being at the apex of the cone of visual rays which projects the view he sees. This apex is called the *station point*. *It always represents the position of the observer's eye.*

17. — If a perspective drawing has been made, the station point is fixed for this particular projection, and the observer in looking at it must place his eye at this point in order to have the drawing appear to him correct. If the eye is not placed exactly at the station point, the perspective will no longer exactly cover the object in space, and under some circumstances will appear much distorted. Just here lies the great defect in the science of perspective. It is the assumption that the observer has but one eye and that this eye remains perfectly stationary while viewing an object. Practically, of course, this is never the case. An object is generally seen with two eyes, which, instead of remaining stationary, are turned directly towards each point in the object as it comes under consideration. Evidently it is impossible to realize this condition of things in representing an object upon a plane surface. If, however, the observer closes one eye, and places the other exactly at the station point, or, better still, if a small hole is punched in a piece of cardboard and placed exactly at the station point, and the observer looks through this, the drawing will appear to him absolutely correct. In a small drawing, unless the eye is a long way removed from the station point, the distortion is so

NOTE [1]. — It will be seen that the *perspective* of an object upon any plane is simply its *conical projection* on that plane, the projectors being *visual rays*. The conical projection of an object differs from the ordinary or orthographic projection in that the projectors, instead of being perpendicular to the plane of projection, all pass through some given point. In a perspective drawing this point is the observer's eye.

slight as not to be noticeable. In looking at a drawing, an observer will naturally place himself opposite the centre, and at least some eighteen or twenty inches away. Thus in making a perspective drawing, if the station point is assumed directly in front of the centre of the plane which receives the projection, and no nearer to it than eighteen or twenty inches, the observer will naturally view the drawing from about the correct point. It is also a good rule to make the drawing no larger than can be included between two horizontal lines drawn from the station point, making an angle of about 60° with one another. This is about the angle that one's vision can easily embrace without turning the head. It will be seen that if a large drawing is to be made, the station point should be assumed correspondingly distant from the plane of the drawing.

18. — From § 15 we see that *the perspective of a point upon any plane is the intersection of the visual ray which passes through the point with the plane.* In Fig. 3 a^p is the perspective of the point a upon the plane M.

19. — A line may be considered as the aggregate of all its points. The perspectives of these points will determine the perspective of the line.

20. — *The perspective of a straight line upon a plane will be a straight line, the extreme points of which are the perspective projections of the extremities of the given line.* All the visual rays which pass through the given line form a plane, the intersection of which with any other plane will be a line whose extreme points are projected by the visual rays through the extremities of the given line. In Fig. 3, $a^p\ b^p$ is the perspective of the line ab on the plane M.

21. — The plane which receives the perspective is called the *picture plane.* Any point, line, or surface which lies in this plane will be its own perspective and will show in its true *size and shape.*

22. — In practice, the *picture plane* (Fig. 4) is assumed to be a vertical one, and corresponds exactly to the *vertical coördinate* of orthographic projections.

23. — The *plane of the horizon* (§ 11) is a horizontal plane passing through the observer's eye (Fig. 4), and corresponds exactly to the *horizontal coördinate* of orthographic projections.

24. — All points, lines, surfaces, and solids in space, the perspectives of which are to be found, are represented by their orthographic projections upon these two planes, and their perspectives are determined from of these projections.

25. — Besides the *picture plane* and the *plane of the horizon* a third auxiliary plane is used called the *plane of the ground* (Fig. 4). This is a horizontal plane upon which the object is supposed to rest. Its position may be assumed at will, above or below the plane of the horizon, according to the nature of the perspective it is desired to make. For an ordinary view it should be assumed about five feet (to the scale of the drawing) below the plane of the horizon, this distance being about the height of a man's eye above the ground. If the plane of the ground is assumed far below the plane of the horizon, the observer (whose eye is always supposed to be in the plane of the horizon, see § 11) will have to look down upon the object and the result will be a bird's-eye view. If it is assumed above the plane of the horizon, the observer must look up in order to see the object, and it will appear to him as though situated on a hill.

26.— In Fig. 4, the station point (SP) is shown in front of the picture plane. The object is seen behind it, resting on the plane of the ground. For convenience the picture plane is generally chosen so as to contain some principal vertical line in the object. This line is called a *line of measures*. In Fig. 5, fh is a line of measures. As it lies in the picture plane it will be its own perspective and show in its true size (§ 21), and dimensions may be laid off directly upon it. It may thus be used as a measure for other parts of the drawing which represent points either behind or in front of the picture plane and which consequently do not show in their true size. This will be explained later. Problem VIII., Chapter II., illustrates the use of the line of measures.

27. — Any plane in the object may be extended until it intersects the picture plane. This intersection will be a *line of measures* for that particular plane. In Fig. 5, mn is a line of measures for the plane $abcd$.

28.— In making a perspective drawing three projections of a point are often used, the *vertical projection* (or orthographic projection upon the vertical co-ordinate or picture plane), the *horizontal projection* (or orthographic projection on the horizontal co-ordinate or plane of the horizon), and the *perspective* (or conical projection, by the visual ray, upon the picture plane). Thus in Fig. 6, a is a point in space, a^V is

its vertical projection, a^H its horizontal projection, and a^P its perspective. [1]

29.— All vanishing points will be projected upon the picture plane and the plane of the horizon in the same way as the point a. As they are always at infinite distances from the station point, their *vertical* and *horizontal* projections will in general be at *infinite distances* from the centre of the drawing and cannot be shown. Their *perspectives*, however, will generally be found *within the limits* of the drawing board.

Rule for finding the perspective of the vanishing point of any system of lines : —

DRAW THROUGH THE STATION POINT AN ELEMENT OF THE SYSTEM, AND FIND WHERE THIS ELEMENT PIERCES THE PICTURE PLANE, § 8 and § 18.

30. — Axioms of Perspective Projection.

a. — The perspectives of all the lines of any system will meet at the perspective of the vanishing point of that system.

b. — The *vanishing trace* of any system of planes will be projected upon the picture plane *as a straight line*, and the *perspectives of all planes belonging to this system will vanish in this line.*

c. — Any line lying in a plane will have the *perspective-of-its-vanishing-point* somewhere in the *perspective-of-the-vanishing-trace* of the plane in which it lies.

d. — The perspective-of-the-vanishing-trace of any plane will pass through the *perspectives-of-the-vanishing-points* of any two lines which lie in it.

e. — The perspective-of-the-vanishing-point of the intersection of two planes, will lie at the intersection of the perspectives-of-the-vanishing-traces of the two planes.

NOTE [1].— The dotted lines in Fig. 6 show the method of finding the perspective of the point a by means of the vertical and horizontal projections of the point and the vertical and horizontal projections of the visual ray. The line drawn through SP^H and a^H represents the horizontal projection of the visual ray. The line drawn through SP^V and a^V represents the vertical projection of the visual ray. The line represented by these two projections pierces the picture plane at a^P. This same construction is shown in Figs. 7 and 8. See Problem I, Chapter II.

31. — The above five axioms are the statements of *facts* which *actually exist* in a perspective projection. They are concrete representations of the purely imaginary phenomena stated in §§ 8 to 14 inclusive.

This distinction between the conditions which really exist in a perspective drawing, and which only appear to exist in space, should be kept clearly in mind by the student. Parallel lines in space do not converge as they recede from the eye. It is a purely imaginary condition. They are just as far apart at infinity as they are at the position of the observer's eye. Their perspective representations, however, really do what the lines in space only appear to do. They really converge as they recede from the observer's eye, until finally they meet at a point; this point being the perspective projection of the vanishing point of the system.

32. — The *trace* of a plane upon the picture plane and the *perspective of its vanishing trace* must not be confused. The former is the line in which the plane cuts the picture plane; the latter is the perspective of the line in which the plane appears to vanish. Thus in Fig. 4, the *trace* of the plane of the ground upon the picture plane is the line VH_1, while the perspective of its vanishing trace is the perspective of the horizon VH. (See § 11 and § 30-*b*.)

33. — It would evidently be very inconvenient were we obliged to use two planes at right angles to one another, as shown in Fig. 6, on which to make the perspective drawing. To avoid this difficulty and to make it possible to work upon a plane surface, the picture plane or vertical co-ordinate is supposed to be revolved about its intersection with the plane of the horizon until the two coincide and form one surface; just as in orthographic projections we suppose the vertical co-ordinate to be revolved about its intersection with the horizontal plane until the two coincide. The arrows (Fig. 6) show the direction in which the revolution is supposed to take place.

34. — To avoid the confusion that might be caused by the overlapping of the two co-ordinate planes, and the consequent mingling of the projections upon them, the two planes, after being revolved to coincide, are supposed to be slid apart in a direction perpendicular to their line of intersection. They will then occupy the position shown in Fig. 7. It is evident that thus sliding the planes apart will in no way affect the relative position of the projections upon them, provided

the corresponding projections of the same point are kept vertically in line.[1]

HPP and VH each represents the intersection of the two planes. HPP should be considered as the *horizontal projection of the picture plane*, while VH should be considered as the *vertical projection* of the *plane of the horizon*. The position of the plane of the ground is represented by its trace on the picture plane VH_1.

35.—The plane of the horizon or horizontal co-ordinate is usually, for convenience, placed above the picture plane or vertical co-ordinate, as shown in Fig. 7. It will be noticed that the relative position of the two planes is the reverse of that generally occupied in ordinary projections.

36.—Fig. 8 shows the position of the planes as ordinarily represented upon the drawing board, HPP, VH, and VH_1 being horizontal lines. HPP and VH may be taken any convenient distance apart. HPP is usually assumed near the top of the board, while VH, for reasons which will be understood later, is drawn across the middle.

The position of VH_1 in relation to VH should be determined according to the nature of the perspective it is desired to make. (See § 25.)

37.—From § 11 and § 30–*b* it will be seen that VH represents the *perspective of the vanishing trace* of all horizontal planes, and from § 12 and § 30–*c* it will also be seen that VH must contain the perspectives of the vanishing points of all horizontal lines.

NOTE [1]. Vertical projections must always be compared with vertical projections, and horizontal projections must always be compared with horizontal projections.

CHAPTER II.

ELEMENTARY PROBLEMS.

38.— The following notation will be found convenient. It has been adopted in the succeeding problems and will be followed throughout the book.

The *picture plane* (or vertical co-ordinate) is indicated by the capital letters PP.

The *plane of the horizon* (or horizontal co-ordinate) is indicated by the capital letter H.

A *point* in space is indicated by a small letter.

The same small letter with an index ^{V}, ^{H}, or ^{P}, indicates its *vertical*, *horizontal*, or *perspective projection*, respectively.

A *line* in space is indicated by a capital letter, usually one of the first letters in the alphabet.

The same capital letter with an index ^{V}, ^{H}, or ^{P} indicates its *vertical*, *horizontal*, or *perspective projection*, respectively.

All lines which belong to the same system may be designated by the same letter, the different lines being distinguished by the subordinates $_1$, $_2$, $_3$, etc., placed after letter.

The *trace of a plane upon the picture plane* is indicated by a capital letter (usually one of the last letters in the alphabet) with a capital V placed before it.

The same letter preceded by a capital H indicates the *trace of the plane upon the horizontal co-ordinate*.

The *perspective of the vanishing trace of a system of planes* is indicated by a capital letter preceded by a capital T.

The *perspective of the vanishing point of a system of lines* is indicated by a small v with an index corresponding to the letter of the lines which belong to the system.

PP = vertical co-ordinate or picture plane.

H = horizontal co-ordinate or plane of the horizon.

H_1 = plane of the ground.

a = point in space.

a^V = vertical projection of the point.
a^H = horizontal projection of the point.
a^P = perspective projection of the point.
A = line in space.
A^V = vertical projection of the line.
A^H = horizontal projection of the line.
A^P = perspective projection of the line.
VS = trace of the plane S upon PP.
HS = trace of the plane S upon H.
TS = perspective of the vanishing trace of the plane S.[1]
v^A = perspective of the vanishing point of a system of lines, the elements of which are lettered A_1, A_2, A_3, A_4, etc.[2]

39. — PROBLEM I. — Figs. 9, 10, 11, 12. — *To find the perspective of a point.*

Let the point be given by its two projections a^H and a^V. SP^H and SP^V are the projections of the assumed position of the station point. Draw the projections (R^H and R^V) of a line passing through SP and a. These will represent the visual ray that passes through the given point. This ray pierces the picture plane at a^P, giving the required perspective (§ 18).

In Fig. 9 the given point lies behind the picture plane and above the plane of the horizon;

In Fig. 10 it lies behind the picture plane and below the plane of the horizon;

In Fig. 11 it lies in front of the picture plane and above the plane of the horizon;

In Fig. 12 it lies in front of the picture plane and below the plane of the horizon.

NOTE [1]. — A plane in space may also be designated by the letters of any two lines which lie in it. Thus the plane AB would be a plane determined by the two lines A and B. TAB would indicate the perspective of the vanishing trace of the plane.

NOTE [2]. — A straight line may be designated by the letters of any two points which lie in it. Thus the line ab would be a straight line determined by the two points a and b. v^{ab} would indicate the perspective of the vanishing point of the line. It is sometimes convenient to use the notation in place of the general one.

40. — Problem II. — Fig. 13. — *To find the perspective of a straight line.*

Let the line be given by its two projections A^H and A^V. The perspective of a straight line will be a straight line, the exteme points of which are the perspectives of the extremities of the given line (§ 20). The perspective of the point a is found at a^P by Problem I. The perspective of b is found at b^P. Thus A^P is the perspective of the line.

41. — Problem III. — Figs. 14, 15, 16. — *To find the perspective of the vanishing point of a system of lines, the projections of one of its elements being given.*

Let A_1^H and A_1^V represent the projections of the given element. Draw the projections (A^H and A^V) of a line parallel to the given element and passing through the station point. This line will belong to the same system as A_1 and must therefore pass through the vanishing point of the system. As it also passes through the station point, it may be considered as the visual ray that projects the perspective of this vanishing point. Thus v^A, the point where it pierces PP, will be the required perspective, § 29, [1].

42. — In Fig. 15 the given system is a horizontal one. The vertical projection (A^V) of the element, which passes through the station point, will coincide with the vertical projection of the horizon, and the perspective of the vanishing point of this system (v^A) will be found upon this line (§ 37).

43. — In Fig. 16, the given system is perpendicular to the picture plane. The vertical projection of the element which passes through SP will be a point and coincide with SP^V. Thus the perspective of the vanishing point of the system will coincide with the vertical projection of the station point.

Note [1]. The perspective projection of any point is the *intersection* of some line with the picture plane (§ 18). This intersection will have a horizontal and a vertical projection, just as does every point in space. It is evident that the *vertical* projection of this intersection always coincides with the intersection itself. The *horizontal* projection will be found on HPP, vertically in line with the intersection. Thus in Figs. 14, 15 and 16, vA is the *intersection* of the picture plane, by the element of the *system A* which passes through the station point. vA is the *vertical* projection of this intersection. Its *horizontal* projection is the point where HPP is crossed by the horizontal projection of the element of the *system A* which passes through the station point.

In making a perspective drawing, the vertical projection of this intersection (vA) is the only one used.

44. — *If the given system is parallel to the picture plane, the perspectives of its elements will be parallel to themselves and to the real lines in space.*

It is evident that an element of such a system, which is drawn through SP, will pierce the picture plane at infinity. Thus the *perspective* of the vanishing point of the system will be at infinity, and must lie upon the vertical projection (page 22, note 1) of the element of the system, which is drawn through SP. The perspectives of all elements of the system (since they are drawn to meet at this point, § 30-a) will be parallel.

All vertical lines are parallel to the picture plane. Thus *the perspectives of the elements of a vertical system will be vertical lines.*

It must not be supposed that systems of lines which are parallel to the picture plane do not apparently converge and meet in a point, just as does every other system of lines seen in perspective. Their perspective projections on the picture plane, however, *being drawn parallel to themselves and to the lines in space*, *actually become elements of the system to which they belong*, and hence appear to the observer to converge just as much as the other elements of the system.

In the case of a system which is not parallel to the picture plane, it is evidently impossible to make perspective projections of its elements actually a part of the system to which they belong. Hence these perspective projections must be drawn converging in order to represent the apparent convergence of the lines in space.

45. — PROBLEM IV. — Figs. 17, 18, 19. — *To find the perspective of the vanishing point of a system of lines, having given the true angle the system makes with the plane of the horizon, and the angle the horizontal projections of its elements make with the picture plane.*

Fig. 17. — Let the system vanish upward and to the right, making an angle of 45° with the plane of the horizon. The horizontal projections of its elements make angles of 30° with PP.

Suppose an element of the system to pass through SP. A^H, making an angle of 30° with HPP, will be its horizontal projection. Where this line pierces PP will be the required perspective of the vanishing point, § 29. If the vertical projection of this element were known, the perspective of its vanishing point could be found as in Problem III. Instead of this projection, however, we have given the true angle which the element makes with H. If the element is

revolved parallel to PP, its vertical projection will show this true angle. Pass a plane (X) through the element, perpendicular to H. It will contain the station point. HX and VX are its horizontal and vertical traces respectively. It is evident that the element (A) will pierce PP at some point in the line VX.[1] This point will be the required perspective of the vanishing point. Now revolve the plane X about VX as an axis, into PP. As the point where A pierces PP lies in this axis, it will not move during the revolution. SP will describe a horizontal arc and be found at SP^H_1, SP^V_1. The horizontal projection of A is now parallel to PP, and its vertical projection will show the true angle which the line makes with H. A line (A^V_1) drawn through SP^V_1, making an angle of 45° with VH, will represent this projection. Where this line crosses VX will give the point (v^A) where A pierces PP. This will be the required perspective of the vanishing point of the system.

46. — Fig. 18 shows the solution of the same problem when the given system vanishes downward and to the right.

47. — Fig. 19 shows the solution of the problem when the elements of the given system lie in planes perpendicular to PP and H, making angles of 45° with H, and vanishing upward.

48. — Problem V. — Fig. 20. — *Having given the projections of an element of a system of lines, to find the perspective of the vanishing point of the system after it has been revolved about a vertical axis through a given angle.*

Let A^H_1, A^V_1 represent the projections of an element of the given system. Suppose the system to be revolved about a vertical axis until the horizontal projections of its elements make angles of $\delta°$ with PP.

As in the preceding problem, imagine an element of the system (in its final position) to pass through the station point. A^H, making an angle of $\delta°$ with HPP, will be its horizontal projection. Where this line pierces PP will be the perspective of its vanishing point. The vertical projection of the element corresponding to this horizontal projection is not known. We do know, however, from the given element, that when its horizontal projection makes an angle of $\alpha°$

Note [1]. Since X contains the station point, VX must be the perspective of the vanishing trace of the plane X (see § 10). Since A lies in this plane, the perspective of its vanishing point must lie in VX (§ 30 — c).

with PP, the corresponding vertical projection makes an angle of $\beta°$ with H. As in Problem IV., pass a plane through A, perpendicular to the plane of the horizon. It will contain the station point. HX and VX are its horizontal and vertical traces respectively. It is evident that the element (A) will pierce PP at some point in the line VX, which will be the perspective of its vanishing point. Revolve the plane (X), containing the element (A) and the station point, about VX as an axis, until it makes an angle of $\alpha°$ with PP. The horizontal projection of A now makes an angle of $\alpha°$ with PP, and we know its corresponding vertical projection makes an angle of $\beta°$ with H. During the revolution, SP has described a horizontal arc and will be found at SP^H_1, SP^V_1. Through SP^V_1 draw the vertical projection of A, making an angle of $\beta°$ with VH. This line will cross VX at v^A, giving the required perspective of the vanishing point of the system.

49.— Most of the foregoing problems are very simple, and their applications need no special explanation.

Problem V. may be considered as the most general case of finding the vanishing point of a system of lines. Problem IV. is a special case under the general one.

In Problem V. *any* two projections of an element of the system may be given, to find the vanishing point of the system after it has been revolved through *any* angle.

In Problem IV. special projections of the element must be given, *i. e.*, the horizontal projection of the given element must always be *parallel to the vertical co-ordinate*, and the vertical projection must consequently show the true angle that the element makes with the horizontal co-ordinate.

An illustration of the application of Problem V. is seen in finding the vanishing point of the diagonal ab of the cube, Problem X., Fig. 24. The plan and elevation of the diagonal of the cube are given, and it is desired to find its vanishing point after the cube has been revolved through a certain vertical angle into the position shown at D.

An illustration of the application of Problem IV. is seen in finding the vanishing points of the sides of the rectangular hole in the case shown in Problem IX.

50. — PROBLEM VI. — Fig. 21. — *To find the perspective of the vanishing trace of a system of planes, one of the elements of the system being determined by two intersecting straight lines, the vertical and horizontal projections of which are given.*

Let B_1^H, B_1^V and A_1^H, A_1^V determine the given element. Through SP draw the projections of two lines parallel to A_1 and B_1, respectively. These two lines will determine the element which passes through the station point. As this element contains the lines A and B, the perspective of its vanishing trace (and hence the perspective of the vanishing trace of the system to which it belongs) will be a straight line passing through the perspectives of the vanishing points of A and B (§ 30-*d*). v^A is the perspective of the vanishing point of A. v^B is the perspective of the vanishing point of B. TAB is the perspective of the vanishing trace of the given system of planes.

51. — PROBLEM VII. — Fig. 22. — *To find the perspective of the vanishing point of the intersection of two planes.*

Let A_1^H, A_1^V and B_1^H, B_1^V determine one of the given planes, and C_1^H, C_1^V and D_1^H, D_1^V determine the other.

The perspective of the vanishing point of the intersection of these two planes will lie at the intersection of the perspectives of their vanishing traces (§ 30-*e*).

Through SP draw lines parallel to A_1 and B_1 respectively. A^H, A^V and B^H, B^V are their projections. v^A is the perspective of the vanishing point of A. v^B is the perspective of the vanishing point of B. TAB is the perspective of the vanishing trace of the plane determined by A and B.

Through SP draw two other lines parallel to C_1 and D_1 respectively. C^H, C^V and D^H, D^V are their projections. v^C is the perspective of the vanishing point of C. As D is parallel to PP, the perspective of its vanishing point will lie on D^V at an infinite distance from SP^V (§ 44). TCD, drawn through v^C parallel to D^V, will pass through this vanishing point and be the perspective of the vanishing trace of the plane determined by C and D. TAB and TCD intersect at v_1, giving the perspective of the vanishing point of the intersection of the two planes.

52.—PROBLEM VIII.—Fig. 23.—*To find the perspective of a rectangular card resting upon the plane of the ground, perpendicular to H and making an angle of 45° with PP, the nearest vertical edge of which is a given distance behind PP.*

The card is shown in plan and elevation at the lower part of the figure. SP^H, SP^V represent the assumed position of the station point. Let VH_1 represent the trace of the plane of the ground. Suppose the card to be placed upon the plane of the ground, in the required position. a_1^H, b_1^H, c_1^H, d_1^H, etc., will be its horizontal projection. Find the perspective of the vanishing point of the upper and lower edges of the card. Since these edges are horizontal lines, their vertical projections will be horizonal lines, no matter what angle the card may make with PP. Hence the vertical and horizontal projections of the edges are known, and their vanishing point (v^A) may be found by Problem III., § 42. v^A will be found upon VH (§ 37).

Imagine the plane of the card to be extended until it intersects PP. This intersection (VY) will be a line of measures (§ 27) for the plane of the card. All points in this line will show in their true position.

If we suppose the line which forms the lower edge of the card to be extended, it will pierce PP at the intersection (n) of VY and VH_1. v^A is the perspective of the vanishing point of this line A^P is the perspective of the line.

In a like manner, if we suppose the line which forms the upper edge of the card to be extended, it will pierce PP at some point (m) in VY. As m is on the line of measures it will be at a distance above n equal to the true height of the card taken from the given elevation (§ 26). A_1^P is the perspective of the upper edge.

The perspectives of the points b and d will be found on A^P, at b^P and d^P respectively. The perspectives of a and c will be found on A_1^P, at a^P and c^P respectively. a^P, b^P, c^P, d^P, is the required perspective of the card.

53.—PROBLEM IX.—Fig. 23.—*To find the perspective of the rectangular hole efgk in the card abcd.*

The perspective of the vanishing points of *ef* and *kg* and of *fg* and *ek* should be found by Problem IV., as indicated in the figure. The given elevation shows the *true angles* that the lines make with the horizontal co-ordinate, and the revolved plan of the card (a, b, c, d)

gives the angle which their horizontal projections make with the picture plane.

Imagine a horizontal line, lying in the plane of the card, to pass through the point f. It will belong to the same system as ac and bd. v^A will be its vanishing point. This line, if extended, will pierce PP at some point in the line VY. As this point is in the line of measures for the plane of the card, it will be as far above VH_1 as the point f is above the lower edge of the card (§ 26). Make on equal $f^v x$. A_2^P will be the perspective of the line. The perspective of the point f will be found on this line at f^P. The perspectives of the sides ef and fg pass through f^P and vanish at v^{ef} and v^{fg} respectively. The perspective of e is found at e^P; of g at g^P.

The perspective of the side ek vanishes at v^{fg}; the perspective of kg at v^{ef}. The points e^P, g^P, and k^P might have been determined in a manner similar to that used for f^P, and the perspective of the rectangular hole might have been found without making any use of v^{ef} and v^{fg}. This method is never so accurate, however, as the one used.

54. — Problem X. — Fig. 24. — *To find the perspective projection of a cube and one of its diagonals, the vertical faces of the cube to make angles of* 30° *and* 60° *with the picture plane.*

Let the cube with its diagonal ab be given in plan and elevation as indicated in the figure.

The revolved plan at D shows the position of the cube in which the perspective projection is to be found.

The first step is to find the perspectives of the vanishing points of the systems ac, ad, and ab. v^{ac} and v^{ad} can be found as were the vanishing points for the upper and lower edges of the card in the last problem. (Fig. 23.)

v^{ab} should be found by Problem V., Fig. 20. The student should carefully follow through the steps taken in Problem V., and apply them to this problem.

The point a lies in the picture plane and on the plane of the ground. Its perspective projection is in VH_1, at a^P.

As af lies in the picture plane, $a^P f^P$ will coincide with af (§ 21) and be, in length, equal to the edge of the cube.

From a^P and f^P draw the upper and lower edges of the nearest right-hand face of the cube, vanishing at v^{ac}.

The perspective of c is found at c^P. The vertical edge of the cube, drawn through c, establishes e^P (§ 44).

From e^P, f^P and a^P, draw edges of the cube, vanishing at v^{ad}.

From a^P draw the diagonal of the cube, vanishing at v^{ab}. The intersection of the diagonal with the edge through e^P establishes the position of b^P.

A line through b^P, vanishing at v^{ac}, should intersect the edge through f^P, and a vertical through d^P, at the same point, completing the perspective projection.

The student is advised to find the perspective of the vanishing point for the diagonal through e and d as a further illustration of Problem V. This will also act as a check upon the accuracy of the work.

CHAPTER III.

METHOD OF REVOLVED PLAN.

55. — PROBLEM XI. — Fig. 25. — *Given the plan and elevations of a house, to find its perspective.*

Preliminary Steps.

56. — The first step in laying out a perspective by this method is to make a *perspective diagram*. This is a plan of the object, showing the position of all external features, such as windows, doors, steps, chimneys, roof-lines, etc., which are to appear in the finished drawing. Having completed the diagram, it is placed at the top of the drawing board as indicated in the figure. It is generally turned at an angle, so that two sides of the object will be seen in the perspective projection. In Fig. 25 the two main walls of the house make angles of 60° and 30° respectively with the picture plane. This position is simply chosen for convenience; the diagram may be turned at any angle according to the view to be shown.[1]

57. — The *picture plane* should be assumed so as to contain one of the principal vertical lines of the house (§ 26). Let it contain the line *ae*. Its horizontal trace (*HPP*) will be a horizontal line passing through the corner of the diagram which represents *ae*.

58. — *VH*, the vertical trace of the *plane of the horizon*, should be assumed about half way between the top and bottom of the drawing board, thus allowing equal space above and below for the perspectives of the vanishing points of oblique lines.

59. — The position of the *plane of the ground* will be determined by the nature of the perspective it is desired to make (§ 25). In the figure it is assumed to be below the plane of the horizon. VH_1 represents its vertical trace.

NOTE [1]. If the diagram is placed so that one of the principal systems of horizontal lines is parallel to the picture plane, the object is said to be in *parallel perspective*. This is a favorite way of making interior perspectives. Exactly the same rules apply, whatever the position of the diagram. See Chapter IV., § 74.

60. — Next assume the *station point*. It must be upon the plane of the horizon (§ 23), and should be in front of the centre of the object (§ 17). Let SP^H and SP^V represent its projections. In this figure and in most of the problems given in these notes, the station point has been chosen much nearer the picture plane than would ordinarily be advisable. This has been done in order that the perspectives of the vanishing points of all systems of lines (except those parallel to the picture plane, § 44) may fall within the rather narrow limits of the plates.

Vanishing Points.

61. — Having assumed the position of the object, the co-ordinate planes, the plane of the ground, and the station point, the next step is to find the perspective of the vanishing point of each system of lines in the object. The construction for this part of the work is drawn in red.

62. — From the given plan and elevations, it will be seen that there are seven different systems of lines the perspectives of which are to be found. They may be grouped as follows: two horizontal systems parallel to ab and cd respectively; two oblique systems which vanish upward, parallel to ef and gh respectively; two oblique systems which vanish downward, parallel to hk and fl respectively; and a vertical system, the perspective projections of which will be parallel to the elements themselves (§ 44).

63. — v^{ab} and v^{cd} may be found by Problem III. The diagram gives the horizontal projections of the lines and, as they are horizontal lines, their vertical projections must be parallel to VH. v^{ab} and v^{cd} are both found on VH (§ 37).

64. — v^{ef} and v^{fl} may be found by Problem IV. The diagram gives the horizontal projections of elements of each system, and the end elevation gives the true angle which they make with the plane of the horizon. It will be noticed that as the two systems make the same angle with the plane of the horizon, but vanish in different directions, v^{ef} is as far above VH as v^{fl} is below it.

65. — v^{gh} and v^{hk} may be found by Problem V. The plan and side elevation give the two projections of an element of each system, and the diagram gives the horizontal projection of an element of each system after it has been revolved through a certain vertical angle to correspond to the position from which the perspective is seen. v^{gh} is found above VH and v^{hk} is found below.

66. — Having found the perspective of the vanishing point of every system of lines in the object, the perspective of the vanishing trace of any plane (*TP*, *TQ*, *TR*, *TS*, etc.) may be found by drawing a straight line through the perspectives of the vanishing points of any two lines which lie in the plane (§ 50).[1]

67. — It will be noticed that v^{gh} falls at the intersection of *TS* and *TP*, v^{cd} at the intersection of *TU* and *VH*, v^{hk} at the intersection of *TR* and *TP*, etc. (§ 51)

68. — Having found v^{ab}, v^{cd}, v^{ef}, and v^{fl}, the perspectives of the other vanishing points in the object might have been found directly from these, in the following way: —

Draw *TS* through v^{ab} and v^{ef}, and *TR* through v^{ab} and v^{fl}.

To find v^{gh}, draw through SP^H the horizontal projection of an element of the system *gh*. This projection will be a line parallel to $g^H\, h^H$ as shown in the diagram. Now suppose a vertical plane to pass through this line. *VX* will be its vertical trace, and the perspective of the vanishing point of *gh* must evidently be somewhere in this trace.[2] As *gh* is a line lying in the plane S_1, the perspective of its vanishing point must lie in *TS* (§ 30–*c*). Hence v^{gh} will be found at the intersection of *TS* and *VX*.

To find v^{hk}, draw *TP* through v^{gh} and v^{cd}. v^{hk} will be found at the intersection of *TR* and *TP* (§ 30–*e*).

69. — The figure formed by the perspectives of the vanishing traces of all the planes in the object, together with *HPP*, *VH*, and the projections of the station point, may be called the *vanishing-point diagram.*

70. — We are now ready to find the perspective projection of the object.

The line *ae* lies in the picture plane. It will therefore be its own perspective and will be a line of measures for the planes *Q* and *U* (§ 26).

NOTE [1]. The perspective of the vanishing point of each system of lines, and the perspective of the vanishing trace of each system of planes in the object, has been found. The student is strongly advised, while making a study of the subject, not to consider any problem complete until the perspective of every vanishing point and every vanishing trace has been determined. This will not only make him familiar with the methods, but will be a check upon the accuracy of his work.

NOTE [2]. As the plane X passes through the station point, VX may be considered as the *perspective of its vanishing trace*, and as *gh* lies in X, *vgh* will be found in VX. (§ 30–*c*.)

It is represented by a vertical line (M) drawn through the point a^H. Where this line crosses VH_1 (at a^P) will be the perspective of the point a. A line drawn through a^P, vanishing at v^{ab}, will represent the perspective of the lower edge of the plane Q. A line drawn through a^P, vanishing at v^{cd}, will represent the perspective of the lower edge of the plane U.

On M lay off the distance $a^P e^P = a^V e^V$. e^P will be the perspective of the point e. From e^P, lines vanishing at v^{ab} and v^{cd}, respectively, will be the perspectives of the upper edges of the planes Q and U. The perspectives of the points w and l are found (as in Problem VIII.) at w^P and l^P respectively. Vertical lines drawn through these points will be the perspectives of the vertical edges of the planes Q and U. From e^P and w^P draw lines vanishing at v^{ef}. These will represent the two edges of the plane S. A line drawn through l^P, vanishing at v^{fl}, will represent the visible edge of the plane R. This line will intersect the line drawn through e^P and v^{ef} at f^P, giving the perspective of the point f. f is one point in the ridge of the roof. A line drawn through f^P, vanishing at v^{ab}, will be the perspective of the ridge.

The plane Q_1 intersects the picture plane in the line M_1. This intersection is a line of measures for the plane Q_1 (§ 27). Where M_1 intersects VH_1 (at y^P) is one point in the perspective of the lower edge of the plane Q_1. The perspective of this lower edge will be a line passing through y^P and vanishing at v^{ab}. On M_1 lay off the distance $y^P z^P$ equal to the height of the plane Q_1. A line drawn through z^P, vanishing at v^{ab}, will be the perspective of the upper edge of this plane. The intersection of the perspectives of the lower edges of the planes U and Q_1 gives the perspective of the point v. The perspective of the point g is found at g^P. Vertical lines drawn through these points will complete the perspective of the plane Q_1.

Lines drawn through g^P and c^P, vanishing at v^{cd}, will represent the upper and lower edges of the plane U_1. The perspective of the point k is found at k^P. $k^P d^P$ will represent the remaining edge of the plane U_1.

A line drawn through g^P, vanishing at v^{gh}, and one drawn through k^P, vanishing at v^{hk}, will determine the plane P. These two lines intersect at h^P. A line drawn through h^P, vanishing at v^{ab}, will be the perspective of the intersection of the planes R_1 and S_1. A line drawn through u^P, vanishing at v^{ef}, will give the intersection of the planes S_1 and U.

To find the perspective projections of the windows and doors in any

plane, proceed as in Problem IX. Imagine horizontal lines to pass through the tops and bottoms of the openings. These horizontal lines will intersect the picture plane in the line of measures for the plane in which they lie. These intersections will show the true heights of the horizontal lines above the ground plane, and may be laid off on the line of measures directly from the given elevation. Having found the perspectives of these imaginary horizontal lines, the positions of the windows, etc., may be determined by projecting from the diagram as shown in the figure, and as illustrated in Problem IX.

The perspective is now complete except the chimney. Imagine some vertical plane in the chimney (U_2, for instance) to be extended until it intersects the picture plane. This intersection (M_2) will be a line of measures for the plane U_2 (§ 27). On this line lay off from VH_1 the true height of the chimney above the plane of the ground, obtaining the point x. Through x draw a line vanishing at v^{cd}. This will represent the upper edge of the plane U_2. The perspectives of m and p are found at m^P and p^P respectively. The perspective of the upper edge of the plane Q_2 will pass through m^P and vanish at v^{ab}. n^P is the perspective of the point n. Vertical lines through m^P, n^P, and p^P determine the vertical edges of the two visible planes of the chimney. The ridge (fo) intersects the plane U_2, at the point o. o^P is the perspective of this point found by projecting from o^H in the diagram. The perspective of the intersection of U_2 and S will pass through o^P and vanish at v^{ef} (§ 30-e). This line will intersect the vertical line through m^P. A line drawn through this intersection, vanishing at v^{ab}, will represent the intersection of Q_2 and S (§ 30-e).

CHAPTER IV.

ROOF LINES AND PARALLEL PERSPECTIVE.

71. — Figs. 26 and 27 give the complete solution of two problems in intersecting roof planes. They contain nothing that is essentially different from the problem given in Chap. III., Fig. 25, and are intended simply as further illustrations.

The perspective of the vanishing point of each system of lines has been found, and the perspective of the vanishing trace of each system of planes determined. To make the diagrams more readily understood, the construction for this part of the work is drawn in red. The vanishing points and vanishing traces have been lettered to correspond with the given plan and elevation, and the student should find little difficulty in following the construction.

In order that the complete diagram for the vanishing points and vanishing traces might be shown upon the plate, the station point, especially in Fig. 27, has been chosen very near the picture plane. If the purpose in view had been to obtain a pleasing perspective projection of the object without regard to the position of the vanishing points, the station point should have been chosen much farther from the picture plane. The unnatural appearance of the object has been further increased by assuming the plane of the ground a considerable distance below the plane of the horizon. This assumption was necessary, however, to show clearly the perspectives of the roof lines.

72. — Fig. 26. — v^{ab}, v^{bc}, and v^{po} were found by Problem III.
v^{de}, v^{ef}, v^{pn}, v^{nq}, v^{rn}, v^{no}, v^{ay}, and v^{zi} were found by Problem IV.
v^{bg}, v^{jk}, v^{uv}, and v^{tx} were found by Problem V.

The systems hc, lm, and rp being parallel to the picture plane will have their perspective projections drawn parallel to themselves (§ 44). These projections will show the *true angles* which the lines in space make with the horizontal co-ordinate.

The perspective of the vanishing trace of any plane must pass through the perspectives of the vanishing points of all lines which lie in it, or which are parallel to it.

73. — Fig. 27. — v^{mn} and v^{sm} were found by Problem III.
v^{xy}, v^{yz}, v^{op}, and v^{pq} were found by Problem IV.
v^{rb}, v^{bg}, v^{sv}, v^{ln}, v^{ib}, v^{bh}, v^{mk}, and v^{wt} were found by Problem V.

The student is advised to follow carefully through the construction of these two figures.

74. — An object is said to be in *parallel perspective* when one of its principal systems of horizontal lines is parallel to the picture plane. Interior perspectives are often represented in this way, the picture plane being assumed coincident with the nearest wall of the room, the remaining three walls being shown in perspective projection.

75. — Fig. 28 gives an example of parallel perspective. The plan and small scale elevation of the object are shown at the top of the plate.

76. — The system of horizontal lines that is perpendicular to the picture plane will vanish at SP^V, § 43. The other principal system of horizontal lines being parallel to the picture plane will have no vanishing point within the limits of the plate. This fact has given rise to the name of *one-point perspective* which is sometimes applied to an object in this position.

The intersection of any horizontal line with the picture plane will show the true vertical height of the line above the plane of the ground, this height being measured from VH_1.

de and *fg* being the perspectives of lines parallel to the picture plane will be drawn parallel to the lines which they represent (§ 44).

77. — There is no essential difference between parallel or one-point perspective and any other kind of perspective. If lines oblique to the picture plane enter into the problem, their vanishing points are found in the usual manner.

78. — There would obviously be many practical difficulties in the way of constructing a complete vanishing-point diagram, as complicated as the one shown in Fig. 27, to the scale at which a perspective drawing is usually made. Such a thing would seldom be done in actual practice. It will often be found convenient, however, to construct a complete vanishing-point diagram to a small scale, the distance between the station point and the picture plane being a certain

factor of the distance to be used in the finished perspective. From this small scale diagram, the position of any vanishing point on the large drawing may be found by multiplying the scale the required number of times.

For illustration, suppose the perspective projection in Fig. 27 is to be made with the station point twenty times as far from the picture plane, as is represented in the figure. Suppose a small vanishing-point diagram is first constructed to the scale shown in the figure. This will be one twentieth of the required scale. Whatever may be the scale, the relations between the various points in the same diagram will remain the same. If it is desired to locate v^{xy} on the large drawing, it may be done by measuring a distance on the horizon to the right of SP^V, twenty times as great as the corresponding distance shown in the small diagram, and, from the point thus established, measuring perpendicularly up from VH, a distance twenty times as great as the distance (v^{sm} v^{xy}) shown by the small diagram. The result will be the position of the desired vanishing point.

This point may very likely fall outside the limits of the drawing board or table on which the perspective is being made. In this case a stool may be so placed that a nail driven into it will establish the position of the required point. The nail will act as a convenient guide for the straight edge when drawing lines to this vanishing point.

It is seldom that in practice the complete vanishing-point diagram has to be made, even at the small scale. Only such vanishing points need be established as will be used in the large drawing.

Considerable attention has been given to the study of the vanishing-point diagram, not only because of its practical use in making a perspective projection, but also because its construction gives the best illustration of the general principles that underlie all perspective drawing.

79.— The scale of a perspective projection depends upon two things: —

First, the scale to which the plan or diagram of the object is drawn.

Second, the relative positions of the diagram and the picture plane. The further behind the picture plane the diagram is placed, the smaller will be the resulting perspective projection.

The further in front of the picture plane the diagram is placed, the larger will be the resulting perspective projection.

80. — Suppose the diagram of an object has been constructed as shown in Fig. 29. The relation between the picture plane and station point has been established as indicated. It is desired to make a perspective projection of the object, with a as its nearest corner, and with the side ab making an angle of 30° with the picture plane. It is also desired to limit the width of the perspective projection to the distance dc.

Draw through SP^H lines passing through d and c. These two lines may be considered as the horizontal projections of the visual rays that project the extreme vertical edges of the object. To fulfil the desired conditions, the diagram must be placed so that it is just included in the angle formed by these two lines, care being taken to keep the required relation between the side ab and the picture plane.

CHAPTER V.

DIRECT METHODS OF DIVISION.

81.—The previous chapters have been devoted to the discussion of the general principles of perspective. Omitting, for the present, the consideration of curved lines and surfaces, these principles will enable the student to solve any problem that may arise, and to make a complete diagram for the vanishing points and traces of the lines and planes involved. In many cases, however, the general methods will be found cumbersome and inflexible. Shorter solutions are needed to save time and perhaps to increase the accuracy of the work. Perspective is full of short cuts and alternative methods of reaching the same result. All these methods are based directly upon the general theory given in the first chapters of the book, and offer, to the student who has mastered the elements of the subject, a very attractive field for study.

The special short solutions that may be devised are infinite in number, and in this brief treatise it will be possible to cover only a very small part of the subject. Without a knowledge of the theory of perspective, the student would be confined to the comparatively few solutions that can be illustrated in the text book. But, understanding the first four chapters, he should find no difficulty in comprehending the principles involved in the problems discussed in this and in the four succeeding chapters, and in adapting these principles to the infinite number of cases that may arise in practical work. With this point of view in mind, typical elementary problems have been selected which are intended to serve more as hints of what may be done than as illustrations of actual cases that are liable to occur in practice. One illustration of the application of these direct methods to a practical example will be found in the problem on the cornice given at the end of Chapter IX.

The most fruitful principles which apply to these short-cut solutions are based upon the methods of direct division, the study of the relations between lines in perspective projection, and the methods of direct measurement.

82. — PROBLEM XII. — Fig. 30. — *Having given the perspective projection of a parallelogram, to find the perspective of its centre and to divide its sides into an even number of equal parts.*

Let $a^P b^P c^P d^P$ represent the perspective projection of a parallelogram, the alternate sides of which vanish at v^{ab} and v^{ac}, respectively. If its diagonals ($a^P d^P$ and $b^P c^P$) are drawn, they will cross at the perspective centre (o^P) of the parallelogram. Lines drawn through o^P vanishing at v^{ab} and v^{ac}, respectively, will divide each of the sides of the parallelogram into two equal parts and will also divide the surface of the parallelogram into four smaller parallelograms having equal areas. By drawing the diagonals of these smaller parallelograms, their perspective centres may be found. Lines drawn through these centres, vanishing at v^{ab} and v^{ac}, respectively, will divide each of the sides of the smaller parallelograms into two equal parts, and each of the sides of the original parallelogram into four equal parts. By continuing this process the sides of the parallelogram may be divided into 8, 16, 32, 64, etc., equal parts.

83. — PROPOSITION. — *If a line which is parallel to the picture plane is divided into any number of parts having any desired proportions, its perspective projection will be divided into an equal number of parts having the same proportions as in the line in space.*

By considering Fig. 31, the truth of the above statement will be made apparent. Let ab represent the line in space, parallel to the picture plane (PP), and divided in any manner by the points c, d, e and f Let $a^P b^P$, parallel to ab (§ 44), be its perspective projection. Let s represent the station point. The visual rays through a and b, together with the line in space, form a triangle, the base of which is divided in a given manner. If through the points of division (c, d, e and f,) lines are drawn meeting at the vertex (s), they will divide any line drawn parallel to the base (as $a^P b^P$) in the same manner as the base. In other words, $\frac{a^P c^P}{a\,c} = \frac{c^P d^P}{c\,d} = \frac{d^P e^P}{d\,e}$, etc. This proportion follows immediately from the similarity of the triangles $sa^P c^P$ and sac, $sc^P d^P$ and scd, $sd^P e^P$ and sde, etc., and as c^P, d^P, e^P, and f^P are the perspective projections of c, d, e and f, the proposition is proved.

84. — PROBLEM XIII. — Fig. 32. — *Given the perspective projection of a parallelogram, two sides of which are parallel to the picture plane, to divide its sides into any number of equal parts.*

Let $a^P b^P c^P d^P$ represent the perspective projection of the parallelogram. $a^P c^P$ and $b^P d^P$ are vertical and hence parallel to the picture plane. The other two sides vanish at v^{ab}.

As $a^P c$ is parallel to the picture plane, any divisions which are laid off on the line in space, of which $a^P_m c^P$ is the perspective projection, will show on $a^P c^P$ in their true proportions (§ 83). Thus $a^P c^P$ may be divided directly into the required number of parts by the points e^P, f^P, g^P, etc. Draw the diagonal $a^P d^P$. Lines drawn through e^P, f^P, g^P, etc., vanishing at v^{ab}, will divide $d^P b^P$ in the desired manner, and will intersect the diagonal ($a^P d^P$) in the points e, f, g, etc , dividing it into a number of equal parts corresponding to those on $a^P c^P$. The line $a^P d^P$, not being parallel to the picture plane, the perspectives to these equal parts will evidently no longer show in their true proportions, but will diminish in length as they approach infinity.

Lines drawn through e, f, g, etc., parallel to the side $a^P c^P$, will divide the two remaining sides ($c^P d^P$ and $a^P b^P$) in the required manner. The lines drawn parallel to the side $a^P c^P$ will be parallel to it in perspective projection since $a^P c^P$ is itself parallel to the picture plane by construction.

85. — If, instead of $a^P c^P$ being divided into a number of *equal* parts, it has been divided in *any manner whatever*, the method of procedure explained in § 84 would enable us to divide $c^P d^P$ into parts having the same proportions as in $a^P c^P$.

86. — It will be noticed that if the diagonal $a^P d^P$ is used, the divisions which fall nearest to c^P on the line $c^P a^P$ fall farthest from c^P on the line $c^P d^P$. If the other diagonal ($c^P b^P$) is used, however (see Fig. 33), the points which fall nearest to c^P on the line $c^P a^P$ will also fall nearest to c^P on the line $c^P d^P$. If the divisions on $c^P a^P$ are not equal, one diagonal may be more convenient to use than the other, according to the nature of the problem.

87. — Problem XIV. — Fig. 34. — *To divide the perspective projection of any line into parts having any given proportions.*

Let a^pb^p vanishing at v^{ab} represent the perspective of the line. Let xy show the manner in which it is to be divided. From a^p, the nearest extremity of the given perspective, drop a vertical line (a^ph). This line may be considered as the perspective projection of some *imaginary* vertical line in space. Any divisions on the imaginary vertical line will show in their true proportions on its perspective projection (§ 83). These proportions are given by the line xy.

From a^p, on a^ph, lay off the distances a^pt_1, t_1s_1, s_1r_1, etc., equal, respectively, to the distances yt, ts, sr, etc., taken from xy. a^px_1 will now represent the perspective projection of some imaginary vertical line in space, which is divided in the same manner as xy.

The scale at which the divisions (a^pt_1, t_1s_1, etc.) are drawn is immaterial. They may be drawn at the same scale as the given line (xy) or at any multiple or fraction of it. It will make no difference in the result, provided the proportions between the divisions remain the same. This is evident, since the true scale of the divisions on the imaginary vertical line in space might be larger or smaller according to the distance of this line from the picture plane (§ 79).

Through the lowest point laid off on a^ph, draw x_1b^p. This may be considered to represent the diagonal of a parallelogram, two sides of which are a^pb^p and a^px_1. One of these sides (a^px_1) is divided in a given manner. The other side (a^pb^p) may be divided in a similar manner by Problem XIII., § 85.

It will be noticed that by using the diagonal x_1b^p, the points of division which are laid off nearest to a^p on the line a^px_1 fall farthest from a^p on the line a^pb^p. Hence the points which are to fall nearest to a^p on a^pb^p should be laid off farthest from a^p, on a^ph; or the parallelogram ($a^pb^pzx_1$) may be completed as indicated by the dotted lines in the figure, and the other diagonal used (§ 86).

88. — If a^pb^p is one side of any parallelogram ($a^pb^pc^pd^p$), the opposite and adjacent sides of the figure may be divided in a manner similar to a^pb^p by applying the principle of Problem XIII.

89. — The line a^ph was drawn *vertical*, as it was supposed to represent the perspective projection of an imaginary *vertical* line in space. The principle of the problem would have held equally well if

a^Ph had been drawn horizontal, or had been given any inclination whatever, for any line that may be drawn may be considered to be the perspective projection of a line parallel to it in space. It will thus be seen that the proposition explained in § 83 will hold, no matter what direction may be given to a^Ph. If a^Pb^P had been more nearly vertical it would have been found more convenient to choose a^Ph more nearly horizontal, as the lines of division would then have crossed it at a greater angle. As a general rule, *it is well to choose a^Ph so that it makes nearly a right angle with the line in perspective that is to be divided.*

90. — Problem XV. — Fig. 35. — *To find the perspective of a dentil course.*

Let $a^Pb^Pc^Pd^P$ represent the perspective projection of the surface on which the dentils are to be placed. From a^P drop a vertical line a^Ph. On this line lay off the spacing of the dentils at any convenient scale. Draw b^Px through the lowest division on a^Ph. a^Pb^P may now be divided in a manner similar to a^Px by Problem XIII. Having divided a^Pb^P in the required manner, the only dimension which remains to be determined is the projection of the dentils. k^P, found from the diagram in the usual manner, will fix this projection. The perspective of the dentil course may then be drawn as shown in the figure.

91. — In a long dentil course, it may be inconvenient to measure off the spacing of all the dentils on a single vertical line. In such a case, the surface on which the dentils are to be placed may be divided into sections, and each section considered separately, as shown in Fig. 36. Through a^P, drop a vertical line as before. Divide it into a number of equal parts corresponding to the number of sections it is desired to make. In the figure, it has been divided into four equal parts. a^Pb^P may now be divided into four equal sections (a^Pc^P, c^Pe^P, e^Pg^P, and g^Pb^P) by Problem XIV. On the vertical line through a^P, measure off the spacing for one quarter of the dentil course (a^Px). This will represent the number of dentils to be found in the first section (a^Pc^P). Through c^P, e^P, and g^P, draw vertical lines (c^Pk, e^Pl, and g^Pm). The lines drawn through the divisions on a^Px, and vanishing at v^{ab}, will divide c^Pk, e^Pl, and g^Pm in a manner similar to a^Px. Draw c^Px, e^Pk, g^Pl, b^Pm. By Problem XIV., a^Pc^P, c^Pe^P, e^Pg^P, and g^Pb^P may each be divided in the required manner.

92. — PROBLEM XVI. — Fig. 37. — *To find the perspective of a flight of steps.*

Let b^Pd^P represent the height of the flight. Let a^Pb^P represent the horizontal length.

Suppose there are five steps from the point a^P to the point d^P. Divide b^Pd^P into as many equal parts as there are steps. Draw a line (a^Pe) through a^P and the first division below d^P. Lines drawn through e, f, g, and h, vanishing at v^{ab}, will intersect a^Pe in the points f_1, g_1, and h_1 and will be the perspectives of the edges of the treads of the steps. Vertical lines drawn through f_1, g_1, h_1, and a^P will be the perspectives of the edges of the risers of the steps. Lines drawn through the intersections of these treads and risers, and vanishing at v^{ac} will represent the intersections of the edges of the steps.

93. — The hints given in Figs. 38, 39, and 40 will be evident to the student without explanation.

CHAPTER VI.

RELATIONS BETWEEN THE STATION POINT AND LINES IN THE PERSPECTIVE PROJECTION.

94. — Problems XII. to XVI., inclusive, will enable the student to divide in any manner whatever the perspective projection of any line, of definite length, of which the vanishing point is known.

The present chapter is devoted to the study of some of the relations that exist between the station point and the perspective projection and between the different lines in the perspective projection itself.

95. — As most of the forms in Architecture are based upon the rectangle, the square, the parallelepiped, or the cube, the study of these shapes will be found most useful in devising short methods. Nearly all architectural forms which are bounded by plane surfaces can conveniently be enclosed in a parallelepiped. Having found the perspective of the enclosing parallelepiped, the perspective of the original form may readily be established within.

96. — PROBLEM XVII. — Fig. 41. — *Given the perspective projection of any two horizontal lines making known angles with the picture plane, to determine the position of the station point.*

Suppose A^P and B^P are the perspective projections of two horizontal lines in space, making angles of 60° and 20°, respectively, with the picture plane. Let VH be as indicated. Since the line A is horizontal, v^A must lie at the intersection of A^P and VH. In a similar manner, v^B must lie at the intersection of B^P and VH.

97. — HPP may be assumed parallel to VH anywhere on the paper (§ 34). From v^A and v^B, draw lines perpendicular to VH, intersecting HPP in the points e and f, respectively. The point e is the *horizontal projection* of v^A (§ 41, foot note). Similarly, the point f is the *horizontal projection* of v^B.

98. — A line (A^H) drawn through e, making an angle of 60° with

HPP, as indicated, will represent the horizontal projection of the element of the system *A*, which projects the vanishing point of this system upon the picture plane. As this element must pass through the station point (§ 29), its horizontal projection must pass through the horizontal projection of the station point.

A line (B^H) drawn through *f*, making an angle of 20° with *HPP*, will represent the horizontal projection of the element of the system *B*, which passes through the station point.

It is evident, therefore, that SP^H must lie at the intersection of A^H and B^H. SP^V will be found on *VH*, perpendicularly in line with SP^H, as indicated in the figure.

99. — A^P and B^P may or may not represent lines lying in the same plane. The true angle they make with one another is shown by the angle between their horizontal projections (A^H and B^H) $= 180° - (60° + 20°) = 100°$.

100. — Problem XVIII. — Fig. 42. — *Given the perspective projection of any horizontal parallelogram, to find from what position it must be viewed in order that it may appear to the observer to represent a rectangle.* (§ 17.)

Let $a^P b^P c^P d^P$ represent the perspective of the horizontal parallelogram. Produce its opposite sides until they meet in v^{ab} and v^{ac}, respectively. Since the parallelogram is horizontal v^{ab} and v^{ac} will both be found on *VH*, as indicated. Assume *HPP*. The point *e* represents the horizontal projection of v^{ab}. The point *f* represents the horizontal projection of v^{ac}. Lines drawn through *e* and *f*, respectively parallel to the systems *ab* and *ac*, will intersect at the horizontal projection of the station point (§ 98), and the angle between these lines will show the true angle that is represented in perspective by $b^P a^P c^P$ (§ 99).

101. — As the angle $b^P a^P c^P$, when viewed from the station point, is to represent a right angle, the angle between the two lines drawn through *e* and *f*, respectively, must equal 90°. The horizontal projection of the required station point will be found at the *intersection* of these two lines. Any two lines, one passing through *e*, the other through *f*, and making 90° with one another, will satisfy the requirements of the problem. As the locus of the intersections of all such pairs of lines is a semi-circle with *ef* as diameter, this semi-circle

will be the locus of all positions of SP^H which will satisfy required conditions. Thus SP^H may have any position on a semi-circle constructed with ef as its diameter. SP^V must be on VH, perpendicularly in line with SP^H. The parallelogram, when viewed from any of the points (SP_1, SP_2, SP_3, etc.) thus represented, will appear to the observer to be a rectangle.

102. — Instead of assuming HPP a separate line, it is generally more convenient to assume it coincident with VH (§ 34), as indicated in Fig. 43. The semi-circle will then have v^{ab} for one extremity of its diameter, and v^{ac} for the other. SP^H may be situated *anywhere* on this semi-circle. SP^V will be on VH perpendicularly in line with SP^H. For convenience, the semi-circle has here been drawn *above* HPP, instead of below, as in Fig. 42. The distance between SP^H and HPP must of course represent the distance of the station point *in front* of the picture plane.

103. — PROBLEM XIX. — Fig. 44. — *Given the perspective projection of any horizontal parallelogram, to find the point from which it must be viewed, in order that it may appear to the observer to represent a square.*

Let $a^Pb^Pc^Pd^P$ be the perspective projection of the given parallelogram, the alternate sides of which vanish at v^{ab} and v^{ac}, respectively. In order that this projection may represent a rectangle, SP^H must be situated somewhere on a semi-circle constructed with v^{ab} v^{ac} as its diameter (Problem XVIII.).

Produce the diagonal (a^Pd^P) of the parallelogram. It will intersect VH in v^{ad}, the perspective of its vanishing point (§ 30, *a* and *c*). Now, in any square, the diagonal bisects the angle between the two adjacent sides. Therefore, if $a^Pb^Pc^Pd^P$ is a square, a^Pd^P must be the *perspective* of the bisector of the angle $b^Pa^Pc^P$, and a line drawn from SP^H to v^{ad} must be the *real* bisector of the angle between the two lines drawn from SP^H to v^{ab} and v^{ac}, respectively (§ 99).

104. — Thus SP^H must lie on a semi-circle, the diameter of which is the distance, on VH, between v^{ab} and v^{ac}, and must lie at such a point on the semi-circle that the line drawn from SP^H to v^{ad} will exactly bisect the right angle formed by the two lines drawn from SP^H to v^{ab} and v^{ac} respectively. SP^V will be found on VH, perpendicularly in line with SP^H.

105.—In order to find the point on the semi-circle where SP^H must be situated, continue the semi-circle to make a full circumference, as indicated in the figure. Draw the diameter mn perpendicular to HPP. Draw a line through n and v^{ad} meeting the semi-circle in the point o. Since an angle inscribed in a circle is measured by *one half* its intercepted arc, the angle formed by the two lines drawn from o to n and v^{ac}, respectively, must equal 45 degrees, or one half the angle formed by two lines drawn from o to v^{ab} and v^{ac} respectively. Therefore SP^H must be at o.

106.—It is evident from the foregoing that the perspective of any horizontal parallelogram may be viewed from such a point that it will represent to the observer a square.

This is a very fruitful proposition, and is not confined to horizontal parallelograms. It will be demonstrated later for any parallelogram (§ 126).

107.—PROBLEM XX.—Fig. 45.—*Given the perspective projection of any horizontal parallelogram, to find the position from which it must be viewed in order that it may represent to the observer a rectangle, the length of the adjacent sides of which are in any given relation.*

Let $a^Pb^Pc^Pd^P$ represent the given perspective parallelogram. Suppose the side ab is to the side ac as 7 : 4. By Problem XIV., divide a^Pb^P into seven equal parts. Through the further extremity of the fourth division draw e^Pf^P parallel to a^Pc^P (vanishing at v^{ac}), forming a square $a^Pe^Pf^Pc^P$.

By Problem XIX., find the position of the station point from which $a^Pe^Pf^Pc^P$ will appear as a square. From this station point the original parallelogram will appear in its proper proportion.

108.—PROBLEM XXI.—Fig. 46.—*To construct the perspective of a horizontal square, having given the vanishing points of its sides and the angles its sides make with the picture plane.*

Let v^{ab} and v^{ac}, respectively, be the vanishing points of the alternate sides of the square. ac is to make an angle of 30° with PP.

Construct a semi-circle with $v^{ab}v^{ac}$ as diameter. From v^{ac} draw a line making an angle of 30° with HPP, and intersecting the semi-

circle in SP^H. Connect SP^H and v^{ab}. The bisector of the angle formed by the two lines just drawn will determine the vanishing point (v^{ad}) of the diagonal of the square.

Through v^{ab} draw any line, and let any length (a^Pb^P) on this line represent the perspective of one side of the square. Through a^P and b^P draw the two sides vanishing at v^{ac}. A line drawn through a^P, vanishing at v^{ad} will represent the diagonal of the square and will determine the position of d^P. d^Pc^P vanishing at v^{ab} will complete the square.

It will be noticed that as the vertical projection of the station point is not needed in the construction, its position has not been lettered.

109. — Fig. 47. — Having drawn the perspective of one square, this may be repeated to cover as large a perspective area as desired.

Let *abcd* represent the perspective of the first square, constructed as in the last problem. Through *b* draw a line vanishing at v^{ad}. This will be the perspective of the diagonal of a second square having the same dimensions as the first. The intersection of the line just drawn with the continuation of *cd* will establish the point *e*. A line through *e*, vanishing at v^{ac}, will intersect the continuation of *ab* in the point *f*, thus completing the second square *bfde*. This construction may be repeated indefinitely, as indicated.

110. — Problem XXII. — Fig. 48. — *To construct the perspective of a horizontal rectangle, the lengths of the alternate sides of which bear any given relation to one another, the vanishing points of the sides being given.*

v^{ab} and v^{ac} are the given vanishing points. *ab* is to make an angle of 60° with the picture plane. *ab* is to *ac* as seven is to three.

By Problem XXI., construct a perspective square, *abed*.

By Problem XIV., divide *ae* into seven equal parts.

Make *ac* equal three of these parts.

abcf will be the perspective rectangle sought.

111. — Fig. 49 shows a second method of solving the problem.

By Problem XXI., construct a perspective square *amcd*.

By § 109 repeat this square in *medf*, and again in *egfh*.

By Problem XIV., divide *eg* into three equal parts, and make *eb* equal one of these parts.

ab is now $2\frac{1}{3}$ times the length *ac*, hence *abck* is the required perspective rectangle.

112. — The results to be deduced from the preceding problems may be summarized as follows: —

113. — By means of these problems the student will be enabled to construct any horizontal square without reference to a diagram.

The square gives a relation of equality between two sets of lines at right angles to one another.

Having constructed a square, the relative dimensions of the alternate sides may be changed in any manner whatever by methods of direct division. Thus the perspective of any desired rectangle can be constructed without reference to a diagram.

114. — Any sort of plane figure composed of straight lines may be enclosed in a rectangle. All lines in the figure which do not already touch the sides of the enclosing rectangle may be extended until they do. The perspective of the enclosing rectangle may be found, and the points of contact between the sides of the enclosing rectangle and the lines (produced if necessary) of the enclosed figure may be established by methods of direct division. The perspective of the enclosed figure may then readily be found.

115. — It will be seen that the perspective projection of any plane figure of known proportions, lying in a horizontal plane, may be constructed directly without using a diagram.

116. — Also, having given the perspective projection of any rectangle of known proportions, the position of the station point may be found, from which it should be viewed, or by means of which the original perspective may be changed in proportions or otherwise altered.

117. — Having established the relation between the lengths of lines at right angles to one another, lying in a horizontal plane, the next step will be to establish relations between the dimensions of these two lines and a third line which is at right angles to the horizontal; in other words, to establish relations between the three dimensions of a right rectangular parallelepiped resting on a horizontal plane.

The most convenient method of accomplishing this is to find first the perspective of a cube, in which, of course, the three dimensions must all be equal. Then, by means of direct division, the relations between these three dimensions may be changed in any manner whatever, obtaining as a result the perspective of any right rectangular parallelepiped resting on a horizontal plane.

118. — In the same way that any plane figure, composed of straight lines, may be enclosed in a rectangle, any solid composed of plane surfaces may be enclosed in a right parallelepiped. Thus, if the perspective of any right parallelepiped can be constructed, the perspective of any plane solid can be found from this by further application of the principles of direct division.

119. — PROBLEM XXIII. — Fig. 50. — *To construct the perspective projection of a right cube resting upon a horizontal plane, the sides of the cube making given angles with the picture plane, and the vanishing points for the horizontal edges being known.*

Let v^{ab} and v^{ac} be the given vanishing points for the horizontal edges.

By Problem XXI., construct a perspective square ($a^Pb^Pc^Pd^P$), the sides of which make the required angles with the picture plane. From a^P, b^P, and c^P, drop vertical lines representing the visible vertical edges of the cube.

The diagonal of the vertical side of the cube, formed on a^Pc^P, will pass through the point c^P and by its intersection with the vertical edge drawn through a^P will determine the length (a^Pf^P) of this edge. Since any face of a cube is a square, the diagonal (f^Pc^P) of the vertical face considered, must make a true angle of 45° with the horizontal co-ordinate.

VX is the vanishing trace of the vertical face considered, and hence must contain the vanishing point of the diagonal f^Pc^P. The position of this vanishing point may be found by adapting Problem IV. The line drawn through SP^H and v^{ac} will represent the horizontal projection of an element of the system to which the diagonal belongs. Imagine a vertical plane to pass through this element. It will contain the station point. Its vertical trace will coincide with VX. Revolve this plane (containing the element and the station point) about VX as an axis, into the vertical co-ordinate. The station point

revolves in a horizontal arc to SP_1^H SP_1^V. The horizontal projection of the element is now parallel to the picture plane and its vertical projection must show the true angle the element makes with H, which equals 45°.[1] Through SP_1^V draw the vertical projection of the element, making 45° with HPP. The intersection of this projection with VX determines the vanishing point (v^{fc}) of the element and hence of the diagonal f^Pc^P. Through v^{fc} and c^P draw this diagonal, establishing the point f^P. The cube may now be completed by drawing lines from f^P, vanishing at v^{ab} and v^{ac} respectively.

120. — It will be seen that in the problem just demonstrated the essential point is to establish the position of v^{fc}. Having established this vanishing point, the diagonal *fc* determines the relation of equality between the vertical and horizontal dimensions.

It will be noticed that as the diagonal of the face of a cube always makes a true angle of 45° with the horizontal co-ordinate, the distance from v^{ac} to v^{fc} will always equal the distance from v^{ac} to SP_1^V, and therefore must be equal to the distance from SP^H to v^{ac}. Thus, after having determined the perspective of the upper face of the cube, the position of v^{fc} may be established immediately, and will be found where a vertical through v^{ac} cuts an arc drawn with v^{ac} as centre and with a radius equal to the distance from v^{ac} to SP^H. This gives a very easy method for constructing a cube.

121. — In Fig. 51 the cube has been constructed as explained in the last paragraph. The perspective of the upper face *abcd* was first found by Problem XXI. The vanishing point (v^{fc}) for the diagonal of the vertical face *acf* was found where the vertical through v^{ac} intersects the arc drawn with v^{ac} as centre and having a radius equal to the distance from v^{ac} to SP^H.

In a similar manner, v^{fb} was found where the vertical through v^{ab} intersects the arc drawn with v^{ab} as centre, and having a radius equal to the distance from v^{ab} to SP^H.

As a check on the accuracy of the work, the diagonals of the two sides (*fac* and *fab*) should meet on the vertical edge drawn through a^P.

NOTE [1]. It is sometimes desirable to find vanishing points for lines lying in the face of a cube, making angles other than 45° with the horizontal. V X must contain the vanishing points of all such lines lying in the face *fac* (Fig. 50) and the vertical projection of any such line, which is drawn through SP_1v must show the true angle which the line makes with the horizontal.

122. — Problem XXIV. — Fig. 52. — *Given the vanishing points of the horizontal edges of a right rectangular parallelepiped resting upon a horizontal plane, to find its perspective, its vertical faces to make given angles with the picture plane, and its three dimensions to have any given relation.*

By Problem XXIII., § 121, construct a cube (*abcdf*) with its vertical faces making the required angles with the picture plane. Suppose the side *an* is to be $\frac{2}{3}$ the height of the parallelepiped, and the side *ak* to be $\frac{5}{6}$ the height. Since *abcdf* is a cube, *ac* equals *af*, equals *ab*. Divide *ac* into 3 equal parts, and make *an* equal two of them. Divide *ab* into 6 equal parts and make *ak* equal to 5. The resulting parallelepiped constructed on *aknm* will have its dimensions in the required relation.

123.— Problem XXV.— Fig. 53.— *Having given the perspective projection of a house, rectangular in plan, the dimensions of the sides being known, to find the position of the station point from which the projection should be viewed, or by means of which additions to the drawing can be made.*

Let the perspective projection be given as in the figure. Suppose, in the actual house, the side *ac* measures 60 feet, and the side *ab* measures 40 feet. Divide *ac* into three equal parts by Problem XIV. Make *ak* equal 2 of these parts. *abkd* will then be the perspective of a part of the floor of the house, which is a square. By Problem XIX., find the point from which *abkd* must be viewed in order to represent a square. This will be the required station point.

124. — If it should be desired to determine the height of the ridge in relation to the dimensions of the sides of the house, construct, by § 121, a cube having for its base the square *abkd*. *abgf* represents one face of such a cube. Compare the distance *ae* with the height (*ag*) of the cube.

125. — The discussion has thus far dealt with vertical and horizontal lines only. The principles deduced, however, may be applied to any two planes at right angles to one another. The problems following in this chapter will serve to illustrate the adaptation of these principles to oblique planes.

126. — Problem XXVI. — Fig. 54. — *Given the perspective parallelogram abcd lying in an oblique plane, to find the position from which it must be viewed in order to represent a square.*

Produce the opposite sides of the parallelogram obtaining v^{ab} and v^{ac}. TM must be the vanishing trace of the plane in which the parallelogram lies.

The vanishing point of ad will be found on TM at v^{ad}.

Imagine a plane parallel to that in which the parallelogram lies, to pass through the station point. This will be the *visual plane* which projects the vanishing trace (TM) on the picture plane (see § 10). Call this visual plane "M".

The visual ray that projects v^{ac} upon the picture plane must lie in this plane M, since it also passes through the station point and is parallel to the line ac. Similarly, the visual ray that projects v^{ab} upon the picture plane must lie in the plane M. Since $abcd$ is a square, these two visual rays must make an angle of 90° with one another. This is evident, since each visual ray belongs to a system of lines, the elements of which are parallel to ab and ac, respectively.

Since each of these visual rays passes through the station point, and since they both lie in the plane M, SP will be found at their intersection. And since they make an angle of 90° with one another, SP must be situated somewhere on a semi-circle drawn in the plane M, with $v^{ab}\,v^{ac}$ as diameter.

Now imagine the plane M (containing the station point) to be revolved about its vanishing trace (TM) into the plane of the paper. v^{ab}, v^{ac}, and v^{ad}, being on the axis of revolution, will not move. The semi-circle constructed with $v^{ab}v^{ac}$ as diameter, on which the station point lies, will, after revolution, show in its true size and shape, as indicated in the figure.

The station point in its revolved position will be found somewhere on this circle, and at such a point that a line drawn from it to v^{ad} will bisect the angle between the two lines drawn from it to v^{ab} and v^{ac}, respectively. (Compare § 105.) This establishes the position of the station point in the plane M, at SP^{M}, and it only remains to revolve this plane back into its original position in order to find the true position of the station point.

As the plane M revolves back about TM into its original position, the station point will describe a circular path, the plane of which must be perpendicular to the axis of revolution (TM). Call this plane X. A line (VX) drawn through SP^{M} perpendicular to TM will represent the vertical trace of this plane.

The radius of the circular path, described by the station point, will be equal to the distance (xy) of the station point from the axis of revolution (TM). When the plane M has reached its original position, the vertical projection of the station point must be found on VH.

VH may be assumed anywhere between the points y and w,[1] and the intersection of VH with VX will establish the position of SP^V.

SP^H must be vertically in line with SP^V, and as far in front of the picture plane as it is in front of VX. If we suppose the plane X to be revolved about its vertical trace (VX) as an axis, into the plane of the paper, the circular path described by the station point will be seen in its true size and shape. The point x (on TM) will be the centre of this circular path, xy will be its radius. A line (HX) drawn from SP^V perpendicular to VX will represent (in its revolved position) the intersection of the plane X with the plane of the horizon. The distance rs will represent the true distance of the station point in front of the picture plane, at the point where its circular path intersects the plane of the horizon. Thus SP^H must be vertically in line with SP^V and at a distance from HPP equal to rs.

When the figure ($abcd$) is viewed from the point in space represented by SP^V, SP^H, it will appear to the observer to be a square, lying in the plane M.

127. — A line drawn through the points s and x will represent the intersection (in its revolved position) of the plane M with the plane X. Therefore the angle sxy is the true angle that the plane M (and hence the plane of the square $abcd$) makes with the picture plane. It will be noticed that the plane M vanishes downward.

128. — A line drawn through s, perpendicular to xs, will represent (in revolved position) a line perpendicular to the plane M and passing through the station point. This line will intersect the picture plane where it crosses VX, giving the vanishing point (v^{af}) of lines perpendicular to the plane M (§ 29).

The sides of a right parallelepiped constructed on $abcd$ as a base must vanish at v^{af}.

NOTE [1]. It is evident that VH must be assumed between the points y and w which limit the circular path described by the station point in its revolution with the plane M. Otherwise, this circular path will not cut the plane of the horizon (§ 11). The meaning of this is that the parallelogram $abcd$ cannot represent a square when viewed from any point above y or from any point below w.

129. — PROBLEM XXVII. — Fig. 55. — *Having given the perspective of the upper face of a cube resting on an oblique plane, to construct the perspective of the cube.*

Let the parallelogram *abcd* be given as the perspective of the upper face of the cube. By Problem XXVI., find SP^V, SP^H, v^{ab}, v^{ac}, v^{ad}, and v^{af}.

Lines through a, b, c, and d, vanishing at v^{af}, will represent the edges of the cube, perpendicular to *abcd* (§ 128).

The next step will be to find the vanishing point for the diagonal of the face N. This vanishing point must lie somewhere on TN.

Imagine the visual plane of the system N, *i. e.*, the one that contains the station point, to be revolved about TN into the plane of the paper. A line through SP^V perpendicular to TN will represent the projection of the circular path described by the station point in its revolution. As the lines *ac* and *af* are at right angles to one another, the station point in revolved position must be found upon a semicircle constructed with v^{ac}, v^{af} as diameter, and therefore at SP^N.

Lines drawn from SP^N to v^{ac} and v^{af}, respectively, will represent (in revolved position) the visual rays that project v^{ac} and v^{af} upon the picture plane. The bisector of the angle formed by these two visual rays will determine v^{ag}.

Draw the diagonal *ag*, determining the length of the side *cg*.

If it is desired, v^{ae} may be found in a similar manner. The diagonal *ae* will determine the length of the side *be*. A line drawn through g, vanishing at v^{ac}, and one drawn through e, vanishing at v^{ab}, should intersect on a line drawn through a, vanishing at v^{af}.

130. — Having constructed the perspective projection of the cube, the relative length of its sides may be altered by methods of direct division, in any desired manner.

131. — If the result of the preceding problem be studied, it will be found to give a comparatively simple method for constructing the complete vanishing point diagram of a cube situated on any inclined plane.

It will be seen that the vanishing traces (TM, TN and TQ) for the three systems of planes in the cube form a triangle. At each apex of the triangle is situated a vanishing point for one of the three systems of lines in the cube. The vanishing point for the intersection of any two faces of the cube will be at the vertex of the triangle

which is opposite to the vanishing trace of the third face of the cube. For illustration: ab is the intersection of the faces M and Q. Its vanishing point (v^{ab}) is found at the vertex of the triangle opposite TN.

Moreover, the vanishing point for the intersection of any two faces in the cube will be found to lie on a straight line drawn through SP^V, perpendicular to the vanishing trace of the third face. If this straight line is continued, it must also pass through the position of the station point, after the latter has been revolved about the vanishing trace of the third plane, into the plane of the paper. For illustration: a line drawn through v^{ab} and SP^V is perpendicular to TN, and also passes through SP^N.

From this symmetry of the figure we may easily construct the complete vanishing point diagram for any cube.

132.—PROBLEM XXVIII.—Fig. 56.—*To construct the complete vanishing point diagram of a cube, having given the vanishing points of the edges of one face.*

Let v^{ab} and v^{ac} be the respective vanishing points for the alternate edges of one face.

Draw TM through v^{ab} and v^{ac}. Construct a semi-circle with $v^{ab}v^{ac}$ as a diameter. SP^M may be chosen anywhere on the semi-circle.[1] The bisector of the angle formed by the two lines drawn from SP^M to v^{ab} and v^{ac}, respectively, will determine v^{ad}.

From SP^M draw an indefinite straight line perpendicular to TM. Assume SP^V on this line. (See foot note, page 55.)

Through v^{ab} and SP^V draw an indefinite straight line, and through v^{ac} draw TN perpendicular to the line just drawn. The intersection of TN and the line drawn through SP^M and SP^V will determine v^{af}.

Construct a semi-circle on v^{ac} v^{af} as diameter. The intersection of this semi-circle with the line drawn through v^{ab} and SP^V will establish the position of SP^N. v^{ag} will be in TN and on the bisector of the angle formed by two lines drawn from SP^N to v^{ac} and v^{af}, respectively.

Draw TQ between v^{af} and v^{ab}. Construct a semi-circle with TQ for a diameter. The intersection of this semi-circle with a line pass-

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NOTE [1]. In a case where the station point is known, SPM must lie at the intersection of the semi-circle with a line drawn through $SP\,V$, perpendicular to TM.

ing through v^{ac} and SP^V will give SP^Q. v^{ae} will be in TQ and on the bisector of the angle formed by two lines drawn from SP^Q to v^{ab} and v^{af}, respectively.

To find SP^H, draw an arc with the point x as centre, and a radius equal to the distance from x to SP^M. The intersection of this arc with a line parallel to TM, through SP^V, will give rs, the distance of SP in front of the picture plane. SP^H will be vertically in line with SP^V at a distance from HPP equal to rs.

133.— Since the perpendiculars from the vertices of a triangle to the opposite sides meet in a common point, it is evident that any three points may represent the vanishing points of three systems of lines at right angles to one another. A triangle may be constructed with its vertices at any three given points. The lines drawn from each vertex perpendicular to the opposite side will intersect at SP^V. SP^H may then be found as in Problem XXVI.[1]

The six vanishing points (v^{ab}, v^{ac}, v^{af}, v^{ad}, v^{ag}, and v^{ae}) together with SP^V and SP^H, may be called a *cubic system*.

NOTE [1]. Since SP^M, SP^N, and SP^Q each represent a revolved position of the same point, it is evident that the figure shown in Fig. 56 must be the development of a triangular pyramid, the base of which is formed by the three lines TM, TN, and TQ, and the apex of which is coincident with the true position of the station point. Also, since the planes M, N, and Q are respectively parallel to the faces of a cube, the apex of the pyramid must be formed by a right triedral angle.

CHAPTER VII.

Method of Cubes.

The principles explained in Problem XXIII furnish a very simple method of constructing the perspective projection of an object directly from given plan and elevations without the use of a revolved plan or diagram.

In Fig. 51, §121, a cube has been constructed making use of the diagonals (of the vertical faces) which vanish upward. In the same way that these vanishing points (v^{fb} and v^{fc}) were found (§120) the vanishing points for the diagonals which vanish downward could have been determined and could have been used in the construction of the cube.

The vanishing point for the diagonal vanishing downward in the right hand vertical face would have been found on a vertical line through v^{ac} and as far below v^{ac} as v^{fc} is above v^{ac}. Its position could have been determined by continuing the arc drawn through v^{fc} and SP^H with v^{ac} as center, until this arc intersected the vertical dropped from v^{ac}.

In a similar way the vanishing point for the diagonal sloping downward in the left hand face of the cube could have been established. It would have been found on a vertical through v^{ab} and as far below v^{ab} as v^{fb} is above v^{ab}, or in other words, where the arc drawn through v^{fb} and SP^H with v^{ab} as center, crossed a vertical dropped from v^{ab}.

This construction is indicated in Fig. 53^A. In this figure v^{ab} and v^{ac} are assumed to be given. SP^H is chosen on a semi-circle constructed with v^{ab} and v^{ac} as base, and in such a position on this semicircle that lines drawn from SP^H to v^{ab} and v^{ac} respectively, will make the desired angles with the picture plane.

A vertical line TL drawn through v^{ab} represents the vanishing trace of the left hand vertical face of the cube. The two points where this vanishing trace is cut by a semi-circle drawn with v^{ab} as center and passing through SP^H, will give the two vanishing points for the diagonals of the left hand vertical face of the cube.

Similarly, a vertical line drawn through v^{ac} will be the vanishing trace of the right hand face of the cube. Where this vanishing trace is cut by a semi-circle constructed with v^{ac} as center and passing through SP^H will give the two vanishing points for the diagonals in the right hand vertical face of the cube.

The vanishing point for one diagonal of the horizontal face of the cube is found at the intersection of VH with the bisector of the angle formed by the two lines drawn from SP^H to v^{ab} and v^{ac} respectively.

The vanishing point for the other diagonal in the horizontal face will generally fall outside the limits of the drawing. It will be found where VH is cut by a line drawn from SP^H, making an angle of 90° with the line from SP^H to v^{dg}.

These eight vanishing points, viz: those for the two horizontal edges of the cube, those for the two diagonals in the left hand vertical face, those for the two diagonals in the right hand vertical face, and those for the two diagonals of the horizontal face, together with the three vanishing traces (TR, TL, and VH) for the three systems of planes in the cube, and with the two projections of the station point constitute what may be called a *Vertical Cubic System.*

Having established the vertical cubic system, the cube may be drawn as indicated in Fig. 53^A. The nearest lower corner (a) may be assumed in any convenient position in accord with the kind of view it is desired to show. From a the lower horizontal edges of the cube may be drawn vanishing at v^{ab} and v^{ac} respectively.

Through a an indefinite vertical line may be drawn representing the nearest vertical edge of the cube. This line is generally assumed to lie in the picture plane and thus becomes a line of measures for the cube. It will be a line of measures not only for vertical heights but for horizontal distances in perspective on ac and ab. Thus the vertical height (ad) of the cube may be laid off directly from a at any convenient scale. This distance is not only the vertical height of the cube but may be considered as the true measure of the lower horizontal edges as well. A line drawn from d vanishing at v^{dc} will determine on the right lower horizontal edge of the cube, a distance ac which is just equal to the measure ad. ad shows in its true length, ac not being parallel to the picture plane will show the perspective of this length.

If it is desired to measure any other distance on ac it may be done in a similar manner. Lay off the true measure of the desired distance on the vertical line of measures through a, as am for illustration. A line through m vanishing at v^{dc} will determine a distance (am_1) on ac equal to the length $a\,m$. am_1 evidently will not show in its true length.

If, again, it is desired to measure on the lower left hand horizontal edge of the cube any distance whatever, as an_1, lay off first the true measure (an) of this distance on the vertical line of measures through a. A line through the point n vanishing at v^{db} will determine the required perspective (an_1).

It will thus be seen that the vertical cubic system is not only useful in the construction of a cube, but may be used to establish directly the perspective dimensions of any object rectangular in plan, the vertical line an being used as a line of measures both for vertical and for horizontal distances.

It will be noticed that the vanishing points of the diagonals which vanish downward are the only ones needed to establish horizontal distances. In the Method of Cubes these vanishing points are given special names and special notation. Thus the vanishing point for the diagonal vanishing downward in the right hand face is known as the *measure point* for the lower right hand horizontal edge (ac) of the cube, and would be lettered in this case m^{ac} as indicated by the red letters.

In a similar way the vanishing point for the diagonal vanishing downward in the left hand face is known as the *measure point* for the lower left hand horizontal edge (ab) and would be lettered in this case m^{ab}.

The system of lines vanishing at m^{ac} are called *measuring lines* for ac, and the system vanishing at m^{ab} are called *measuring lines* for ab.

The vertical line an is a *line of measures* for ac, ab, and vertical distances.

The vanishing points for the diagonals vanishing upward in the vertical faces are not given any special names and are designated as usual by the letters of the lines in the object which vanish to them.

PROBLEM, Fig. 53^B. *To find by the Method of Cubes the perspective projection of the object shown in plan and elevations.*

The first step in making a perspective by the method of cubes is to construct a *vertical cubic system* as just explained. This has been done as shown in the figure.

Instead of constructing the perspective projection directly it is sometimes convenient to construct first a complete plan of the object, in perspective lying in an auxiliary horizontal plane below the plane on which the object is supposed to rest. Each point in this plan will lie vertically below the corresponding point in the perspective projection, and having constructed the perspective plan, the perspective projection may be found directly from it.

Such a plan has been constructed in Fig. 53^B in order to avoid as much as possible confusion in the construction lines. In practice this is not generally necessary, and the perspective projection may be drawn at once.

VH_1 the vertical trace of the plane of the ground has been chosen as indicated. VH_2 the vertical trace of the auxiliary horizontal plane on which the perspective of the plan is to be drawn, may be chosen at any convenient distance below VH_1.

The point a for the nearest corner of the plan has been chosen on VH_2 as indicated. From a lines drawr to v^{ac} and v^{ab} respectively will represent the two nearest edges of the plan.

The line of measures has been drawn vertically through a.

On this line from the point a, lay off the distance ab_1 equal to the distance ab as taken from the given plan. A measuring line through b_1 vanishing at m^{ab} will determine the poin† b on the perspective plan. From b the right rear edge of the plar vanishes at v^{ac}.

From a on the linc of measures, lay off the distance ac_1 equal to ac in the given plan. A measuring line through c_1 vanishing at m^{ac} will determine the point c on the perspective plan. From c the left rear edge of the plan vanishes at v^{ab} and meets the right rear edge in the point p.

Mark off on the line of measures by the points j_1, u_1, t_1, and s_1 distances equal to aj, ju, ut, and ts taken directly from the given plan. Measuring lines through these points vanishing at m^{ab} will determine the points j, u, t, and s on the perspective plan.

In a similar manner establish on ac the points x, w, v, and c.

The complete perspective plan may now be drawn as indicated, except that the projection (ar) of the porch is as yet unknown. This projection may be found as follows: — On the line of measures lay off *downward* from the point a the distance ar_1 equal to ar taken from the given plan. A measuring line through r_1 vanishing at m^{ac} will determine on ca produced the point r in the perspective plan. From r, the front edge of the porch vanishes at v^{ab}.

It will be noticed that measurements for all parts of the perspective plan are referred to the two lines ac and ab. This must be done in order to preserve the scale throughout the drawing. For instance, the point g is not found directly, but is established by the intersection of the two lines drawn through x and j respectively. These two latter points being measured on ac and ab. Again the point k is not found directly but is determined by first establishing the points j and v on the lines ab and ac respectively, and then drawing lines from these two points parallel to the sides of the plan, which determine k by their intersection.

It will also be noticed that distances on ac *behind* the point a are laid off *above* the point a on the line of measures, while distances on ac which fall *in front* of the point a are laid off *below* the point a on the line of measures. The same would of course be true regarding measurements on ab.

Having constructed the perspective of the plan of the object, each point in the perspective projection will be vertically over its corresponding point in the plan. Starting with the nearest vertical edge, the point a^P will be determined by measuring from VH_1 the true height of this edge as taken from the elevation.

The points $c^P b^P g^P k^P h^P$, etc., will be found vertically over their corresponding points in perspective plan.

It will generally be found convenient to establish auxiliary lines of measures to determine the heights of the different parts of the perspective projection. Thus to find the height of the main ridge, produce the line nh in the perspective plan, till it intersects the picture plane, as determined by its intersection with VH_2. A vertical line drawn through this intersection will be a vertical line of measures for the ridge, the distance *1–2* showing the true height of the ridge above the ground.

Similarly an auxiliary line of measures may be found for the ridge of the porch, by producing the line *te* to its intersection with VH_2, and erecting a vertical line from this intersection. The distance *3–4* will show the true height of the porch ridge above the ground.

By a little ingenuity vanishing points may be found for many of the sloping roof lines.

Thus the line *gh* lies in the system of planes that vanishes to the right and makes a true angle of 45° with the horizontal, vanishing upward. It is therefore the diagonal of a square constructed on *gk* as base and must vanish at v^{gh} as shown.

Similarly, *hk* must vanish at m^{ac}.

The vanishing point for *de* must lie in TS and will be found where TS is met by a line drawn through SP_1^V sloping upward and making an angle of 30° with the horizontal (see foot note page 52).

In a like manner v^{ef} will be found on TS where TS is met by a line drawn through SP_1^V sloping downward and making 30° with the horizontal.

The line $a^P g^P$ is the intersection of the planes M and P. Its vanishing point is therefore at the intersection of the vanishing traces of these two planes. TM must pass through v^{ab} and v^{gh} and is thereby established. TP can also be determined. One point (v^{ac}) in TP has already been found. A second point on TP may be found as follows: — Assume a vertical plane through the ridge to intersect the plane P. This intersection will be shown by the line *yz* in the perspective projection. The line *yz* is parallel to the plane S and must therefore have its vanishing point in TS. Since the plane P makes a true angle of 45° with the horizontal (as shown by the elevations) the line *yz* must also make a true angle of 45° with the horizontal. It is then parallel to the diagonal of a vertical square constructed in the plane S and must vanish at v^{yz}. Therefore TP must pass through v^{ac} and v^{yz}. The intersection of TM and TP will determine v^{ag}.

As $c^P k^P$ is the intersection of the planes P and N, v^{ck} will be found at the intersection of TP and TN. This point falls outside the limits of the paper.

Similarly v^{bm} will be found at the intersection of TM and TO. This point also falls outside the limits of the paper.

CHAPTER VIII.

DIRECT MEASUREMENT OF LINES IN PERSPECTIVE.

134. — The three preceding chapters have dealt with the relative measurements of lines in perspective, Chapter VII. developing a method of constructing a perspective projection directly from given plan and elevations. A second and somewhat similar method of constructing a perspective without reference to a diagram is described in this chapter and the one following. The method is based upon the principles of *direct measurement.* These principles are of wide application and their developement is shown in the following discussion.

135. — Fig. 57. — Suppose $a^H b^H$ to represent the horizontal projection of a horizontal line. Assume the position of the station point to be as indicated by SP^V and SP^H. $a^P b^P$ is the perspective of the line, vanishing at v^{ab}.

Produce $a^H b^H$ till it intersects HPP in the point c^H. c^P on $a^P b^P$ produced, will be the perspective of this point and show where ab pierces the picture plane. VH_1 drawn through c^P, parallel to VH, will represent the vertical trace of the plane which contains the line ab.[1]

On HPP lay off $c^H r^H$ equal in length to $c^H b^H$, and draw $r^H b^H$. $c^H r^H b^H$ will be an isosceles triangle, one side of which ($c^H r^H$) lies in the picture plane.

The vanishing point for $r^H b^H$ is found at m^{ab} in the usual manner.

Through b^P, draw a line vanishing at m^{ab}, and intersecting the picture plane in the point r^P. $c^P r^P b^P$ will be the perspective of the triangle $c^H r^H b^H$. As one side ($c^P r^P$) of this perspective triangle lies in the picture plane, it will show in its true length, and as the triangle is isosceles, $c^P r^P$ will be a measure of the true length of $c^P b^P$.

NOTE [1]. The *vertical trace* of any plane must be parallel to the *perspective of the vanishing trace* of the system to which it belongs, since these two lines are the intersections of parallel planes with the picture plane. Also, when the system is perpendicular to the picture plane, the perpendicular distance between the vertical trace of any plane and its vanishing trace will show the true distance between that plane and the visual plane of the system to which it belongs.

If, through a^H, a line ($a^H s^H$) is drawn parallel to $b^H r^H$, $s^H r^H$ will be equal to $a^H b^H$. $a^P s^P$, vanishing at m^{ab}, is the perspective of $a^H s^H$, and $s^P r^P$ will be the measure of the true length of $a^P b^P$.

If d^P is any point on $a^P b^P$, the true length of $a^P d^P$ and of $d^P b^P$ may be found by drawing through d^P a line ($d^P w^P$) parallel to $r^P b^P$, *i. e.*, vanishing at m^{ab}. $s^P w^P$ will be a measure of the true length of $a^P d^P$. $w^P r^P$ will be a measure of the true length of $d^P b^P$.

136. — The line $a^P b^P$ may be divided in any manner whatever by laying off on $s^P r^P$ the true lengths of the required divisions (*1*, *2*, *3*, etc.), and drawing through these divisions lines vanishing at m^{ab}. The intersections of these lines with $a^P b^P$ will establish the required divisions.

137. — The vertical trace (VH_1) of the plane containing *ab* is a *line of measures* for *ab*, and for all other lines lying in that plane (§ 26).

138. — The system of lines vanishing at m^{ab} is called the system of *measure lines* for *ab*.

139. — The vanishing point (m^{ab}) of this system of measure lines is called the *measure point* for *ab*.

140. — Since a construction similar to that in Fig. 57 could be made for any line parallel to *ab*, and since for all such lines the measure point would coincide with m^{ab}, the *measure lines* and the *measure point* for the line *ab* are also the *measure lines* and the *measure point* for the whole *system ab*.

The *line of measures* (§ 137) for any element of the system *ab* will be the vertical trace of the particular plane which contains that element.

141. — The measure point for any system of lines will be designated by a small letter *m*, with an index corresponding to the index of its related system. Thus, m^{ab} signifies the measure point of the system vanishing at v^{ab}.

142. — Fig. 57-*a*. Instead of laying off the distance $c^H r^H$ in the same direction as $a^H b^H$, as was done in Fig. 57, $c^H r^H$ might have been laid off in the opposite direction, as shown in Fig. 57-*a*. The discussion in regard to Fig. 57 will apply equally well to Fig. 57-*a*. Thus the system *ab* may have two measure points situated on opposite sides of the vanishing point of the system.

143. — *Every system of lines in perspective has two measure points, the positions of which bear a fixed relation to the vanishing point of the system.*

It will be remembered that the triangle $c^H b^H r^H$, in Figs. 57 or 57-a, was by construction isosceles. In finding the vanishing points of the sides of the triangle, the line from SP^H to e, was made parallel to $c^H b^H$, and the line from SP^H to f was made parallel to $r^H b^H$. Therefore, the three lines joining the points f, e, and SP^H must also form an isosceles triangle, and the distance from f to e must equal the distance from SP^H to e. As the distance from f to e equals the distance from m^{ab} to v^{ab}, it follows that the *measure point* of the system ab is as far from the *vanishing point* of the system ab as SP^H is from e.

Now e is the horizontal projection of v^{ab} (see foot note, page 22), and since v^{ab} and SP both lie in the plane of the horizon, the true position of each must be coincident with its horizontal projection; therefore, the line from SP^H to e shows the *true distance* of the station point from the vanishing point of the system ab. In other words, *the measure point for the system ab is as far from the vanishing point of the system ab as that vanishing point is from the station point.*

144. — It is evident that a construction similar to that in Figs. 57 or 57-a can be made, with a similar result, for any line lying in a horizontal plane.

Therefore, *the two measure points for any system of horizontal lines will be found on VH, one on each side of the vanishing point of the system, and as far from that vanishing point as the vanishing point is from the station point.*

In general, but one measure point will be needed. It will usually be found more convenient to use the measure point which is on the side of SP^V, opposite to that of its related vanishing point, as shown in Fig. 57.

145. — Having once determined the relation that exists between the vanishing point of a system and its measure points, the construction shown in Figs. 57 and 57-a can be condensed, as indicated in Fig. 58. *HPP* and *VH* have been assumed coincident. The points f and m^{ab} will then coincide, as well as the points e and v^{ab}. The two measure points for the system ab will be found on *VH*, one at each of the points where this line is cut by a circumference drawn with v^{ab} as centre, and with a radius equal to the distance between v^{ab} and SP^H (§ 144).

146. — PROBLEM XXIX. — Fig. 59. — *To find the true length of any horizontal line from its given perspective projection, the vertical trace of the plane in which it lies, and the projections of the station point, being known.*

Let $a^P b^P$ represent the given perspective projection of the line. Let VH_1 represent the vertical trace of the plane in which the line lies, and let the position of the station point be as indicated.

Continue $a^P b^P$ to its intersection with VH, obtaining its vanishing point v^{ab}. With v^{ab} as centre, and radius equal to the distance between v^{ab} and SP^H, draw an arc intersecting VH in the measure point (m^{ab}) for ab (§ 145).

Through a^P and b^P draw lines vanishing at m^{ab}. The distance intercepted on VH_1 by these lines will show the true length (ab) of the line represented in perspective by $a^P b^P$.

147. — If $c^P d^P$ represents the perspective of some other line belonging to the same system as ab, and lying in the same horizontal plane, its true length may be found in a manner similar to that of ab by drawing measure lines through c^P and d^P. The intercept (cd) of these lines on VH_1 will give the required true length.

148. — If it is desired to measure how far in front of the picture plane the point c^P really lies, it may be done in the following manner. A horizontal line through c, perpendicular to the picture plane, will measure this distance. The perspective of such a line will pass through c^P and vanish at SP^V (§ 43), intersecting the picture plane at the point e^P. The measure point of this line will be found at m^{ce} (§ 145). $c_1 e^P$ will show the true length of $c^P e^P$ and, consequently, the true perpendicular distance of the point c, in front of the picture plane.

149. — The line cd intersects the picture plane in the point g, part lying in front of the picture plane and part lying behind. The true length of the part behind the picture plane is measured from the point g, in one direction, while the true length of the part in front is measured from g in the opposite direction.

150. — It will always be found that the true length of any line of a system, lying *behind* the picture plane, is measured from the intersection of that line with the picture plane, in a direction *from the measure point towards the vanishing point.*

The true length of any such line lying in *front* of the picture plane is measured from the intersection of that line with the picture plane, in a direction *from the vanishing point towards the measure point.* The truth of these statements will be made evident by an examination of Fig. 60.

151. — PROBLEM XXX. — Fig. 61. — *To construct the perspective projection of a horizontal line of known length, making a given angle with the picture plane, the nearest end of the line to be a given distance behind the picture plane.*

Assume HPP and VH coincident. SP^H and SP^V represent the projections of the station point. Let VH_1 represent the vertical trace of the plane in which the line lies.

Through SP^H draw the horizontal projection of the line, making the required angle with HPP. v^{ab} will be the vanishing point of the line. Its measure point will be found at m^{ab}.

SP^V will be coincident with the vanishing point of a system of lines perpendicular to the picture plane, m^{ca} being a measure point for this system.

Through any point (c) on VH_1 draw a line vanishing at SP^V. On VH_1 lay off to the right (§ 150) from the point c, a distance ca_1 equal to the given distance between the picture plane and the nearest end of the line whose perspective is to be found. Through a_1 draw a measure line vanishing at m^{ca} and establishing the perspective (a^P) of the nearest end of the required line. From a^P the line will vanish to v^{ab}.

Through a^P draw a measure line vanishing at m^{ab} and intersecting VH_1 at a. From a, lay off to the right (§ 150) the true length (ab) of the required line. A measure line through b, vanishing at m^{ab} will determine b^P. a^Pb^P will be the required perspective.

152. — PROBLEM XXXI. — Fig. 62. — *To construct the perspective of any desired rectangle lying in a horizontal plane.*

Let v^{ab} and v^{ac}, respectively, represent the vanishing points of the alternate sides. SP^H and SP^V may be determined by Problem XVIII., § 102. m^{ab} and m^{ac} may be found by § 144. a^Pc^P and a^Pb^P may be found by Problem XXX. Lines drawn through b^P and c^P, vanishing respectively at v^{ac} and v^{ab}, will complete the required rectangle.[1]

153. — It is evident that the construction shown in Fig. 58 (§ 145) can be applied to the visual plane of any oblique system, as well as to the visual plane (H) of the horizontal system. Thus measure points may be found for any system of lines.

NOTE [1]. In case vab and vac are so far apart that it would be inconvenient to construct a semi-circle with its diameter equal to the distance between these points, the positions of SPH, SPV, mab and mac may be determined as follows: —

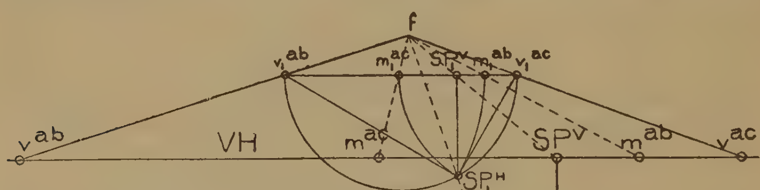
154. — Fig. 63 illustrates the application of the principle of finding measure points adapted to a system of lines lying in oblique planes.

HPP and *VH*, and the projections of the station point, are given as indicated in the figure. Let *TM* represent the vanishing trace of a system of oblique planes. The vertical trace (*VM*) of the visual plane (*M*) of that system will coincide with *TM*.

In order to see the construction for the measure points, the visual plane *M* must be revolved about its vertical trace (*VM*) into the plane of the paper. All lines and distances on this plane will then show in their true shape and size. Since the station point lies in *M*, it will revolve with *M*. Its path will describe the arc of a circle, and will be projected on *PP* in a straight line through SP^V, perpendicular to the axis of revolution (*VM*). In revolved position, the perpendicular distance from the station point (SP^M) to *VM* will show the true perpendicular distance of the station point in front of *VM*, *as measured in the plane M.*

Suppose v^{ab} to represent the vanishing point of any system of lines parallel to the plane *M*. As v^{ab} is on *VM*, its position has not been affected by the revolution of the plane *M*. Since both v^{ab} and *SP* lie in the plane *M*, the true distance between these two points is shown

Through v^{ab} and v^{ac}, respectively, draw two lines meeting at any point (*f*) and forming the triangle v^{ab}, v^{ac}, *f*.

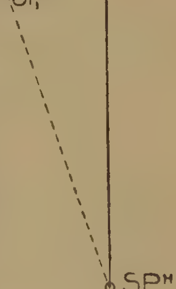


Draw any line ($v_1{}^{ab}$, $v_1{}^{ac}$) parallel to *VH*, forming the triangle $v_1{}^{ab}$, $v_1{}^{ac}$, *f*. The two triangles thus formed are similar.

Treat $v_1{}^{ab}$ and $v_1{}^{ac}$ as though they were the given vanishing points, and find $SP_1{}^H$, $SP_1{}^V$, $m_1{}^{ab}$ and $m_1{}^{ac}$ as indicated.

Lines drawn from the point *f*, passing through $SP_1{}^V$, $m_1{}^{ab}$ and $m_1{}^{ac}$, respectively, will determine by their intersections with *VH* the positions of SP^V, m^{ab}, and m^{ac}, respectively. A line from *f*, through $SP_1{}^H$ will determine by its intersection with a vertical through SP^V, the position of SP^H.

The truth of this construction is evident from the similarity of the triangles involved.



The hint given here is of wide application, but its use necessitates extreme accuracy in draughting, as any error in the small figure will be multiplied in the large one.

by the line drawn from v^{ab} to the revolved position of the station point (SP^M). Therefore, the measure points for the system ab will be found at the two points where TM is cut by a circumference drawn with v^{ab} as centre and passing through SP^M.

If VM_1 represents the vertical trace (see foot note, page 59) of any plane belonging to the system M, and a^Pb^P represents the perspective projection of a line lying in that plane, a^Pb or a^Pb_1 will measure the true length of a^Pb^P.

155. — The statement in § 144 may be adapted to a general case as follows: —

The two measure points for any system of lines will be found on the vanishing trace of the system of planes in which the lines lie, and will be as far from the vanishing point of the system as that vanishing point is from the station point.

156. — TO FIND THE MEASURE POINTS FOR ANY SYSTEM OF LINES, FIND THE TRUE DISTANCE BETWEEN THE STATION POINT AND THE VANISHING POINT OF THE SYSTEM. WITH THIS TRUE DISTANCE AS A RADIUS, AND THE VANISHING POINT OF THE SYSTEM AS A CENTRE, DESCRIBE A CIRCUMFERENCE. THE INTERSECTIONS OF THIS CIRCUMFERENCE WITH THE VANISHING TRACE OF THE PLANE IN WHICH THE LINES LIE WILL DETERMINE THE MEASURE POINTS FOR THE SYSTEM.

157. — The *line of measures* for any particular line of the system will be the vertical trace of the particular plane in which that line lies. This line of measures will always be parallel to the vanishing trace which contains the measure points (see foot note, page 59).

158. — Since any line may be the intersection of two or more planes, it follows that a line may lie in a number of planes at the same time. In fact, an infinite number of planes may be conceived to intersect on a single line. Thus through the vanishing point of any system of lines we may draw the vanishing traces of an infinite number of planes in which the lines may lie. Therefore, any system of lines may be said to have an infinite number of measure points, two being on each vanishing trace that can be drawn through the vanishing point of the system. The locus of all these measure points will be a circle drawn with the vanishing point of the system as centre, and with a radius equal to the true distance of this vanishing point from the station point. Care must be taken to use the proper lines of measures with any particular set of measure points (§ 157).

159. — PROBLEM XXXII. — Fig. 64. — *To find the true length of any line shown in perspective projection, having given the vanishing point of the line, the vertical trace of a plane in which it lies, and the projections of the station point.*

Let $a^P b^P$ represent the perspective of the line, its vanishing point being at v^{ab}. VM_1 represents the vertical trace of some plane containing the line. SP^V and SP^H represent the projections of the vanishing point.

Through v^{ab} and SP^V draw R^V, representing the vertical projection of the distance between v^{ab} and SP. Through e and SP^H draw R^H, representing the horizontal projection of the distance between v^{ab} and SP. The true length of this distance is found by revolving R^H parallel to the picture plane. The corresponding vertical projection (R_1) then shows the required true length.

With R_1 as radius and v^{ab} as centre, draw a circle. Draw TM through v^{ab} parallel to VM_1, establishing by its intersection with the circle the two measure points (m^{ab} and ${m_1}^{ab}$). The true length of $a^P b^P$ is shown at $a^P b$.

160. — As a^P, in Fig. 64, represents the vertical trace of $a^P b^P$, it is evident that any line drawn through a^P would represent the vertical trace of some plane containing $a^P b^P$, and that a line parallel to this vertical trace, drawn through v^{ab}, would represent the vanishing trace of the plane containing $a^P b^P$, and would determine, by its intersections with the circle already drawn, the measure points to be used. Thus, if VN_1 is drawn through a^P to represent the vertical trace of some plane containing the line ab, TN drawn through v^{ab} parallel to VN_1 will determine by its intersection with the circle the two measure points to be used for ab when VN_1 is used for the line of measure.

CHAPTER IX.

METHOD OF PERSPECTIVE PLAN.

161.—The method of perspective plan is based upon the direct measurement of lines in perspective. No diagram, such as that employed in the method of revolved plan (Chapter III.), is used. Instead of the diagram a perspective plan of the object is made directly from the given plan by the principles explained in Chapter VII.

162. — As generally constructed, this perspective plan is supposed to lie in some auxiliary horizontal plane below the plane of the ground on which the object rests. For convenience in establishing a vertical line of measures some principal corner of the plan is ordinarily assumed to lie in the picture plane.

163. — Every point in the perspective plan will be found vertically in line with the corresponding point in the perspective projection of the object. Thus, having constructed the perspective plan, the position of all vertical lines in the perspective projection of the object can be determined immediately.

164.—It is often convenient to construct several of these perspective plans on different auxiliary horizontal planes, one below the other, representing different horizontal sections, through the object (see § 186, Chapter IX., for illustration). Corresponding points in all the perspective plans will be found vertically in line with one another.

165. — Fig. 65 represents the solution of a simple problem by the method of perspective plan. The plan and elevations from which the perspective is to be made are shown in the figure. v^{ab} and v^{ac} have been assumed as indicated. As the given plan is rectangular, SP^H will lie on a semi-circle constructed on $v^{ab}v^{ac}$ as diameter. The position of SP^H on this semi-circle will be determined by the angles which the sides of the plan are to make with the picture plane.

Measure points for the two systems, *ab* and *ac* will be found at m^{ab} and m^{ac}, respectively.

VH_1 is the vertical trace of the plane on which the object is to rest.

VH_2 is the vertical trace of the plane in which the perspective plan is to be constructed. Let one point (a) in the plan lie in the picture plane. Assume the position of this point on VH_2 at a_2.[1] From a_2 the sides of the perspective plan will vanish to v^{ab} and v^{ac}, respectively.

From a_2 lay off on VH_2, a_2b_2 equal to the true length of ab, taken from the given plan. A measure line through b_2, vanishing at m^{ab}, will establish the point b_2^p in the perspective plan. The perspective of the side ac may be found in a similar manner.

166. — It is often convenient to refer all other lines in the perspective plan to the two sides ab and ac. For instance, the distance af, taken from the given plan, may be laid off on VH_2, to the right, from the point a_2, obtaining f_2. A measure line through f_2, vanishing at m^{ab}, will establish the point f_2^p, on $a_2\,b_2^p$ produced. The perspective of de will be found upon the line drawn through f_2^p, vanishing at v^{ac}. To establish the position of the points d_2^p and e_2^p on this line, lay off on VH_2, from a_2 to the left, the distances a_2g_2 and a_2h_2, taken from the corresponding distances on the given plan. Measure lines through g_2 and h_2, vanishing at m^{ac}, will determine the points g_2^p and h_2^p on $a_2c_2^p$. Lines through these two points, vanishing at v^{ab}, will establish the points d_2^p and e_2^p.

167. — Having constructed the perspective plan, or such a part of it as may be needed, every point in the perspective projection of the object will be found vertically above its corresponding point in the perspective plan, as indicated.

168. — The line a^px^p is a vertical line of measures for the planes of the vertical sides of the house.

If the vanishing points of oblique lines are desired, they should be found in the usual manner, as, for instance, v^{yz}.

169. — The method of perspective plan is well adapted to the uses of the landscape architect, who frequently has occasion to make a perspective bird's-eye view of the grounds of some large estate. A complete perspective plan of the grounds may be laid out, including the perspective plans of such buildings as may exist. The perspective projections of these buildings, which are usually small in comparison

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NOTE [1]. It is well to assume the position of $a2$ so that the centre of the perspective plan will not be far out of line with SPH, thus bringing the assumed position of the observer's eye somewhere nearly in front of the centre of the drawing.

with the area of the grounds, may be drawn directly on their perspective plans.

170. — In such a problem, it is frequently convenient to enclose the plan of the grounds in a rectangle, chosen so that two of its opposite sides may be parallel to the picture plane, as indicated in Fig. 66. The enclosing rectangle may be subdivided into smaller rectangles of any desired size, to aid in constructing the lines of the perspective plot, many of which may have to be drawn freehand.

171. — The construction of such a plot is very simple. One side of the enclosing rectangle being assumed to lie in the picture plane, all dimensions on it will show in their true size. SP^V will coincide with the vanishing point for the sides of the rectangle perpendicular to the picture plane. In Fig. 66 the side ab is supposed to lie in the picture plane. The side ac vanishes at SP^V. m^{ac} and m_1^{ac} are measure points for the system ac.

172. — To establish the exact perspective (d^P) of any point (d), first determine the co-ordinates de and df of the point on the given plot. On VH_1, lay off b_1f_1 equal to bf as shown in given plot. d^P will be found somewhere on a line drawn through f_1, vanishing at SP^V. Lay off b_1e_1 equal to be taken from the given plot. A measure line drawn through e_1, vanishing at m^{ac}, will determine e^P. A line drawn through e^P, parallel to VH_1, will establish the required perspective of the point d.

173. — Problem XXXIII., Fig. 67. — *To find the perspective of a cornice from the given plan and elevation.*

This problem is given as another illustration of the application of the methods of direct measurement and perspective plan.

The plan and elevation of the cornice are given as shown in the figure. The angles the sides of the cornice are to make with the picture plane are indicated in the given plan by the angles between PP^H and the running lines of the cornice. The vanishing points for these running lines have been chosen at v^{ce} and v^{cd} respectively. SP^H has been determined in accordance with the data. m^{cd} is a measure point for the system cd.

Imagine a vertical plane to bisect the angle formed by the two sides of the cornice. This plane may be called the *mitre plane*. It will make angles of 45° with the running lines of each side of the cornice, and its projection on the given plan will coincide with the line $c_1^Hc^H$.

This mitre plane will intersect the two vertical planes from which the cornice projects, in the vertical line a_1b_1. If we imagine a series of horizontal lines lying in the mitre plane, to be drawn through the points in the given cornice lettered c, g, h, k, l, m, n, etc., they will intersect the line ab in the points shown in elevation at c_1^V, g_1^V, h_1^V, k_1^V, l_1^V, etc. The vanishing point for these imaginary horizontal lines will be found at v^{cc_1}.

If the line ab is supposed to lie in the picture plane, the perspectives of the points c_1, g_1, h_1, k_1, etc., will coincide with the points themselves. Therefore, the perspective of the line ab may be drawn immediately, as indicated. It will be a vertical line, with the point a_1^P showing at its true height above the plane of the horizon, and with the points a_1^P, c_1^P, g_1^P, h_1^P, k_1^P, l_1^P, m_1^P, etc., showing the true distances between them, as taken from the given elevation.

Through the points a_1^P, c_1^P, g_1^P, h_1^P, k_1^P, l_1^P, m_1^P, etc., thus established, draw lines vanishing at v^{cc_1}, representing the perspectives of horizontal lines lying in the mitre plane and passing through these points. The perspective (c^P) of the point c will be found somewhere on the line drawn through c_1^P, vanishing at v^{cc_1}; the perspective (g^P) of the point g will be found somewhere on the line drawn through g_1^P, vanishing at v^{cc_1}; the perspective (h^P) of the point h will be found somewhere on the line drawn through h_1^P, vanishing at v^{cc_1}, etc.

Let VH_2 represent the vertical trace of some auxiliary horizontal plane. Produce the line $a_1^Pb_1^P$ until it meets VH_2 in the point b_2^P. A line drawn through b_2^P, vanishing at v^{cc_1}, will represent the perspective plan of the vertical mitre plane. m^{cc_1} is a measure point for horizontal lines lying in this plane. VH_2 is the corresponding line of measures.

On VH_2 lay off to the left from b_2^P the distance $c_1^Hc^H$ taken from the plan and representing the true horizontal distance (measured in the mitre plane) which the point c projects in front of the line ab. Through the point c_2 thus obtained, draw a measure line vanishing at m^{cc_1}. This line will establish the position of c_2^P, which is the perspective plan of the point c in the cornice.

A vertical through c_2^P will intersect the line drawn through c_1^P, vanishing at v^{cc_1}, and determine the perspective projection (c^P) of the upper point in the angle of the cornice.

In a similar manner find g^P, h^P, k^P, l^P, m^P, etc., may be found, determining the complete intersection of the mitre plane with the face of the cornice, or, in other words, the complete *mitre section* of the cornice.

Lines drawn from each point in the face of this mitre section, vanishing at v^{ce} and v^{cd}, respectively, will be the perspectives of the horizontal edges of the mouldings in the cornice, and will complete (with the exception of the dental course) the required perspective projection.

174. — It will be seen that the complete mitre section was determined directly from the given plan and elevation by means of two lines of measures, — one, a vertical line of measures (ab) on which vertical distances were established; the other, a horizontal line of measures (VH_2) from which horizontal distances were determined.

175. — In finding the perspective of the dentil course, the outer faces of each of the two sets of dentils were supposed to form one continuous vertical plane. The plane formed by the faces of one set will intersect the plane formed by the faces of the other set, in an imaginary vertical line, indicated in the given plan by w^H. The perspective plan of this line will be found at $w^P{}_2$. A line drawn from $w^P{}_2$, vanishing at v^{cd}, wil be the perspective plan of the vertical plane, determined by the front faces of the dentil course that vanishes towards the right. m^{cd} wiil be a measure point for horizontal lines lying in this plane.

Project the point $w^P{}_2$ back to the line of measures by a measure line vanishing at m^{cd}, obtaining the point *0*. From this point, lay off on VH_2, to the right, the points *1*, *2*, *3*, *4*, *5*, etc., representing the true spacing of the dentils as taken from the given plan. Measure lines through these points, vanishing at m^{cd}, will determine the corresponding perspective plan of this spacing, which must lie on the line drawn through $w^P{}_2$, vanishing at v^{cd}. Vertical lines drawn through the points thus obtained will establish, by their intersections with a line through $w^P{}_1$, vanishing at v^{cd}, the corresponding perspective spacing of the front faces of the dentils. From this spacing the perspective of the dentil course may readily be constructed.

The two lines drawn through w^P, vanishing at v^{cd} and v^{ce} respectively, form the two alternate sides of a horizontal perspective parallelogram, the diagonal of which is represented by a line through w^P, vanishing at v^{cc_1}. Having found the spacing for the dentils on the first side of the parallelogram, the spacing may be found on the second side by the principle explained in §85, as indicated.

176. — The positions of the dentils might also have been established by the method explained in Problem XV., Chapter V.

CHAPTER X.

CURVES.

177. — Up to this point in the discussion, only straight lines have been considered. As the perspective projection of a straight line upon a plane surface is always a straight line, two points are sufficient to determine its direction. In the preceding problems, one of these points has usually been the perspective of the vanishing point of the system to which the line under consideration belonged.

178. — In most cases, the perspective projection of a curved line will be a curve. It evidently cannot be determined by two points. Either of the three following methods may be used to find the perspective of a curved line: —

1. — The line may be treated as a series of points, the perspective projection of the line passing through the perspective projections of all the points of which it is composed (§ 19). A few points chosen at convenient distances along the line will usually determine its perspective projection with sufficient accuracy. In Fig. 68, A^H and A^V are the projections of a curved line. a^P, b^P, c^P, etc., are the perspectives of the points a, b, c, etc. A^P, passing through a^P, b^P, c^P, etc., is the perspective of the line.

2. — If the curve is of simple, regular form, it may be inscribed in a polygon. The perspective of the polygon may then be found. A curve inscribed within this perspective polygon will be the perspective of the given curve. Fig. 69 shows a general case of this method of solution. The points of tangency (a, b, c, d) are points on the circle, and the sides of the enclosing polygon give the directions of the curve at these points. Additional points on the perspective curve may be established by drawing the diagonals of the polygon and finding the perspectives of the points where these diagonals are cut by the curve. Also, the sides of the enclosing polygon may be increased in number, each additional side giving one additional point (the point of tangency) on the curve, and the direction of the curve at this point. To avoid the necessity of finding new vanishing points, the directions of

these sides should, if possible, be chosen parallel to some of the prin- pal lines in the object, the vanishing points for which have already been found.

3. — If the curve is very irregular it can be enclosed in a rectangle, as shown in Fig. 70, and the rectangle divided into any number of smaller rectangles by lines drawn parallel to its sides. The perspective of this rectangle and its dividing lines may then be found and the perspective of the given curve established.

179. —The circle is perhaps the form of curve most frequently met in architectural work. Its perspective will be the intersection of the picture plane with a cone of visual rays, which has the given circle for its base. It will thus be seen that the perspective of a circle may take the form of *any conic section*. In Figs. 71, 72, 73 and 74, suppose the line *ab* to represent the given circle seen as though viewed in the direction of its plane. Let *PP* represent the picture plane, and *s* the position of the station point. *asb* will then represent the cone of visual rays which projects the perspective of the circle *ab*.

The perspective of a circle may be considered under one of the following five cases: —

a. — If the plane of the given circle is parallel to the picture plane, as shown in Fig. 71, its perspective will be a *circle*, the *centre* of which will be the *perspective of the centre* of the given circle. The radius of the required perspective may be determined by finding the perspective of any point on the given circle. If the three circles in the figure are equal and equally distant from the picture plane, it is evident that their perspectives will be three circles of equal radii.

b. — If the given circle lies in the plane of the horizon, as shown in Fig. 72, its perspective projection will be a *straight line*.

c. — If the picture plane cuts the cone of visual rays, parallel to an element of the cone, as shown in Fig. 73, the perspective of the circle will take the form of a *parabola*.

d. — If the picture plane *cuts the given circle*, but in such a manner that it is not parallel to an element of the projecting cone, as in Fig. 74, the perspective of the circle will be a curve having the form of an *hyperbola*.

e. — In other cases the perspective of the circle will be an *ellipse*.

This is the form which the perspective of a circle most frequently takes.

180.—Fig. 75. In general, the perspective of a circle is most conveniently found by the second method given in § 178. The enclosing polygon will be a square, and should in general be chosen so that two of its sides are parallel to the picture plane. If the circle is a horizontal one, the remaining two sides will then vanish at SP^V, and a measure point (m^{ab}) for these sides will be found on VH, as far from SP^V as the station point is from the picture plane (§ 144). This measure point, in this particular case, will coincide with the vanishing point for one diagonal of the square.

In the figure, one point (k) in the circle lies in the picture plane, hence a_1c_1 must show the true length of the side of the enclosing square. a_1b_1 shows the true length of the side ab. b^P may be determined by a measure line through b_1, vanishing at m^{ab}.

Since m^{ab} is also the vanishing point for the diagonal of the square, the perspective of the square might have been determined by drawing the perspective diagonal of the square through a_1, vanishing at m^{ab}, and establishing the point n^P.

The perspectives (d^P and g^P) of the points where the circle cuts the diagonal of the enclosing square may be established as indicated, by the auxiliary circle constructed on ef as a diameter.

181.—In Fig. 76, the circle, the perspective of which is to be found, is situated behind the picture plane. The distance o_1n_1 is the true measure of the distance of the centre of the circle from the picture plane.

The distance o_1a_1 shows the true length of the radius of the circle. a_1c_1 shows the true length of the sides of the enclosing square. Two lines drawn through a_1 and c_1 respectively, and vanishing at v^{ab}, will be the perspectives of two sides of the square. The diagonal of the square will be drawn from the point n_1, vanishing at m^{ab}, and establishing the points a^P, o^P, and d^P.

The perspective of the enclosing rectangle and the circle may now be drawn.

Since a^Pc^P is parallel to the picture plane, all divisions on it will show in their true *relative* (though not actual) sizes (§ 83). Therefore, the perspectives of the points where the circle cuts the diagonals of the enclosing square may be found from the auxiliary circle, constructed with a diameter equal to a^Pc^P, as indicated.

182.—The perspectives of the given circles in Figs. 75 and 76

must be ellipses (§ 179). It will be seen by an inspection of the figure that the perspective of the centre of the circle cannot coincide with the intersection of the axes of the ellipse. If the perspective of any diameter of the circle is drawn through o^P, the perspective of the half of the diameter that is the farther from the picture plane must be the smaller, but if o^P coincided with the intersection of the axes of the ellipse, both halves of the perspective diameter would have to be equal. It is possible to determine the axes of the ellipse which forms the perspective of a circle, but as the method presents few practical advantages over the one given here, and as, after all, the true perspective projection of a circle is not often used, the method is not given in these notes.

183. — In Fig. 77 is shown the perspective of a series of vertical semi-circular arches lying in a plane oblique to the picture plane. It will be noticed that the crown (a) of the arch is not the highest point in the perspective of the arch. The perspective of some point (b) nearer the picture plane than the crown of the arch will be the highest point in the perspective projection. It is evident that this will always be true when the eye of the observer is assumed to be below the crown of the arch.

If the eye is assumed to be above the crown of the arch, the highest point in the perspective will be the perspective of some point farther from the picture plane than the crown of the arch, Fig. 78.

If the eye is on a level with the crown of the arch, the perspective of the crown will be the highest point in the perspective, Fig. 79.

The points h^P and k^P, in each arch, have been established by means of an auxiliary quadrant. The construction will be evident.

184. — With the exception of the straight line, the perspective of a sphere may assume any of the forms taken by the perspective of a circle; for the perspective of a sphere is really the perspective projection of the *circular section*, along which the visual rays are tangent to the sphere.

This perspective projection will be a circle only when the sphere is in such a position that the visual ray through its centre is perpendicular to the picture plane.

In this position the plane of the circular section, along which the visual rays are tangent, will be parallel to the picture plane (§ 179-*a*).

With the sphere in any other position the plane of the circular section of tangency will be oblique to the picture plane, and its perspective will take the form of some conic section other than the circle. See Fig. 86.

185. — There are two general methods for finding the perspective projection of any solid bounded by curved surfaces.

1. — Determine, by methods of descriptive geometry, the vertical and horizontal projections of the line on the surface of the solid that represents the locus of all the points of tangency of the visual rays. The perspective of this line will be the perspective outline of the solid.

2. — Assume the object to be cut by a series of parallel planes. Find the intersection of these planes with the surface of the solid. Find the perspective of each intersection. A line enclosing all the perspectives thus determined will be the perspective outline of the object.

186. — The second method is particularly well adapted to solids of revolution. By assuming the series of parallel planes perpendicular to the axis of such a solid, their intersections with the surface of the solid will all be circles. Fig. 86 illustrates this method applied to a solid of revolution, the contour of which is shown on the right. The solid is supposed to be cut by a number of horizontal planes, each one of which will intersect the surface of the solid in a circle. These planes are indicated on the given contour by the horizontal lines, *bc*, *kl*, *mm*, *op*, etc.

The perspectives of the circles, cut from the solid by the horizontal planes, have been found by a method similar to that explained in § 180. A line enclosing all these perspective circles, and tangent to the circumference of each, will be the perspective outline of the given solid.

CHAPTER XI.

SHADOWS IN PERSPECTIVE.

187. — Shadow is the obstruction of light. If any object is lighted from a single source, the rays of light which fall upon it will be interrupted and a portion of space extending indefinitely behind the object will be deprived of light. This darkened space is called the *invisible shadow* of the object and will take the form of a cone or cylinder, the elements of which are the rays of light tangent to the object.

188. — The intersection of this invisible shadow with any plane is called the *visible shadow* of the object upon the plane. When the term shadow alone is used it usually means the visible shadow.

In Fig. 81, O represents an object in space lighted by solar rays coming in the direction indicated by the arrow. The sun being at so great a distance, no appreciable error will be made if its rays are assumed to be parallel. Thus the invisible shadow of the object will take the form of a cylinder (C). The intersection of this cylinder with any plane (P) will be seen as the visible shadow (S) of the object upon the plane.

189. — If the object is reduced in size until it becomes a single point, its invisible shadow will become the *single ray of light* which has passed through the point. Thus the *shadow of any point upon a plane is where the ray of light through the point pierces the plane.*

190. — The invisible shadow of a straight line will be a *plane* made up of all the rays of light which pass through the line. Thus the shadow of a straight line upon a plane, being the intersection of two planes, will be a straight line. In Fig. 82, $a^s b^s$ represents the shadow of the line ab upon the plane P. The shadow of any point (d) in the line, will be where the ray of light through the point crosses the shadow of the line.

191. — In perspective it is more convenient to deal with lines than with points. In finding the shadow of a point, the shadow of some line passing through the point is first found, and the shadow of the point determined on this line.

192. — In architectural work the sun is usually taken as the source

of light. As the rays from it may be considered parallel (§ 188), they form a system of lines, the vanishing point of which may be found in the usual manner, from the projections of any element of the system.

193. — Fig. 83 shows the solution of a simple problem in perspective shadows. *ab* and *bc* represent the perspective projections of two intersecting lines in space, resting upon the plane of the ground. *ab* is oblique both to the picture plane and the plane of the horizon. The perspective of its vanishing point has been found at v^{ab}. *bc* is a vertical line passing through the point *b* and piercing the plane of the ground at *c*. The perspective of its vanishing point is vertically above *SP* at infinity. The shadows of these two lines fall partly on the plane of the ground and partly on the plane *deg* which forms one side of the triangular prism shown in perspective. The perspective of the vanishing point of *de* has been found at v^{de}, of *dh* at v^{dh}, and of *eg* at v^{eg}.

194. — The assumed direction of the ray of light which passes through the station point is shown by its two projections. v^s represents the vanishing point for the rays of light.

195. — The rays of light which pass through *ab* form a plane, the intersection of which with the ground will be the shadow of *ab* upon that surface (§ 190). As this plane contains the ray of light, its vanishing trace must pass through v^s, and as it contains the line *ab* its vanishing trace must also pass through v^{ab}. *TR* represents this vanishing trace. *VH* represents the vanishing trace of the plane of the ground. The intersection of *VH* and *TR* will be the perspective of the vanishing point for the intersection of the plane of the ground and the plane of the invisible shadow of *ab* (§ 30-*e*), or, in other words, the perspective of the vanishing point for the visible shadow of *ab* upon the plane of the ground. an^s is this visible shadow.

196. — From n^s, the shadow of *ab* leaves the plane of the ground and falls upon *deg*. The vanishing trace of *deg* (*TT*) passes through v^{eg} and v^{de}. The intersection of *TT* and *TR* will give the perspective of the vanishing point for the visible shadow of *ab* on *deg*. The shadow of the point *b* will be found at b^s where the ray of light through *b* crosses the shadow of *ab* (§ 190).

197. — In a similar manner the shadow of *bc* may be found. The perspective of the vanishing trace of the plane determined by the ray of light and *bc* must pass through v^s, and as the perspective of the vanishing point of *bc* is at an infinite distance above the plane of the horizon, this trace must be parallel to *bc*, *i. e.*, a vertical line through

v^s (TP). The intersection of TP and VH will give the perspective of the vanishing point for the shadow of bc and the plane of the ground. Part of the shadow falls on *deg*. The perspective of the vanishing point for this shadow is at the intersection of TP and TT. This shadow should meet the shadow of ab at b^s.

198. — The shadows of *de* and *eg* are found in a similar manner. *eg* being a horizontal line casting its shadow upon a horizontal plane, its shadow will be found to be parallel to itself, *i.e.*, vanishing at v^{eg}.

199. — Fig. 84 is given as a further illustration of shadows in perspective. With the exception of the vanishing point for the shadow on the arm of the cross on the plane S,[1] all the vanishing points and vanishing traces needed for the construction of the figure are shown. The problem will be sufficiently clear to the student without explanation.

200. — The shadow of a curved line may easily be found by enclosing the curve in some sort of a polygon, finding the perspective shadow of the polygon, and inscribing a curve in the perspective shadow thus found. The resulting curve will be the required perspective shadow.

201. — If the rays of light emanate from any source other than the sun, they cannot be considered parallel without error. They will all

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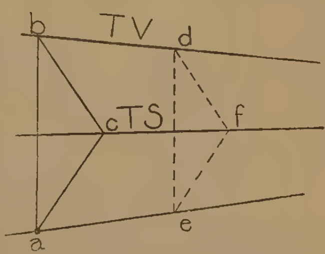
NOTE [1]. This vanishing point falls outside the limits of the plate, at the intersection of TV and TS. A line may be drawn through any point meeting TV and TS at their intersection in the following way: Let TV and TS be as represented in the accompanying sketch. Let a represent any point through which it is desired to draw a line meeting TS and TV at their intersection. Through the point a draw any two lines intersecting TV and TS in the points b and c, respectively. Complete the triangle abc.



From any point (d) on TV draw a line parallel to bc, intersecting TS in f. Through f draw a line parallel to ca. Where this line intersects a line through d parallel to ba determines the point e. From the similarity of the triangles bca and dfe, it is evident that a line drawn through a and e, and the two lines TV and TS, must all meet at the same point. The farther apart the points b and d can be taken, the more accurate will be the construction.

diverge from the luminous point which is their source. The perspective of this luminous point may be found, and we may then proceed exactly as though it were the vanishing point of a system of parallel rays.

If the source of light happens to lie behind the observer, we shall meet the somewhat unusual problem of having to find the perspective of a point that the observer could not see without turning around and looking in the opposite direction (§ 17). This perspective is needed only as a construction point, however, and is really used to determine the direction of the rays of light that lie *in front* of the observer.

202.—If the object is a complicated one, or if the shadows fall upon curved surfaces, it becomes a very long and difficult matter to determine the shadows directly in perspective. In such a case the shadows may first be found on the plan and elevation of the object. The perspective projections of the bounding lines of these shadows may then be determined.

CHAPTER XII.

APPARENT DISTORTION.

203. — It was stated on page 9 of the introduction, that an object in space is exactly represented by its perspective projection. In other words, no distortion can exist in an accurate perspective. Notwithstanding this statement, very disagreeable effects and very *apparent* distortion are often noticed in perspective projections, the accuracy of which can hardly be put in question.

204. — Take for illustration the curious results often seen in a photograph. With a properly constructed lens, the camera, in expert hands, can be used to illustrate all the phenomena of perspective, and to produce, in the photograph, a perfect perspective projection. Fig. 85 illustrates the relation between the photographic projection and the perspective projection as ordinarily constructed. The line (ab) represents the object in space. The optic centre (o) of the lens corresponds to the position of the station point. The sensitive plate PP corresponds to the picture plane. The image which is projected upon this plate is a conical projection of the object in space, the apex of the cone of projectors being at the point o. The plane of the horizon is an imaginary horizontal plane passing through the point o. The relation between the point o and the sensitive plate represents the assumed relation between the observer's eye and the picture plane.

205. — It will be noticed that in the photographic projection the station point (o) occupies a position between the picture plane and the object in space, while as ordinarily assumed in constructing a perspective projection by hand, the picture plane lies between the station point and the object in space. The relation between the object in space, the station point, and the picture plane, which exists in the photographic projection, results in the inversion of the image on the sensitive plate. It is as though the picture plane P_1P_1, Fig. 85, had been revolved from its usual position, about a horizontal axis parallel to itself and passing through the point o, until it reached a position perpendicular to the plane of the horizon, on the side of the

station point opposite to that which it originally occupied. This revolution is indicated by the dotted line in the figure.

The inversion of the image on sensitive plate is of course corrected in viewing the photograph by simply turning it over. This accomplishes the same result as though the sensitive plate had been revolved back about the axis through o into the usual position for the picture plane.

206. — The rays of light from the object, which pass through the point o, proceed in a straight unbroken line, and the cone $a^P o b^P$ is exactly similar to the cone $a^P_1 o b^P_1$. In other words, it is as though the perspective had first been constructed in the usual manner on the plane $P_1\,P_1$, and then this perspective with its cone of visual projectors had been revolved about an axis through the station point, as just explained. If the lens is properly constructed, the photographic projection will be a true perspective. Thus, with a suitable lens, the use of the camera may be a legitimate means of illustrating any of the phenomena in perspective.

207. — A few photographs of simple objects will be sufficient to show what is meant by apparent distortion. Consider Fig. 86. It is supposed to represent a number of perfect spheres all of the same size. The photograph certainly does not convey such an impression to the observer. The centre figure is the only one that can be said to present the appearance of a sphere. No one ever looked at a sphere in space and received the impression of such an egg-shaped mass as that at the extreme right or the extreme left of the photograph. With such results as are seen in this photograph, it is difficult to make the casual observer believe the statements made at the beginning of the chapter. He naturally concludes that, either the perspective projection is not a correct one, or that a perspective projection is not an exact representation of the object in space.

208. — As a further illustration, consider the photograph shown in Fig. 87. This represents a series of cylinders and circular plinths, all the objects of one kind having exactly the same dimensions. If such a series of objects in space were viewed with the eye, the group nearest the observer, *i. e.*, the centre one in the row, would seem to be the largest, while the others would appear to be smaller and smaller as they were situated farther and farther from the observer. The exact opposite is true of the projections in the photograph. The projection of the group at the centre, which is the one nearest to the observer, is the smallest, and as the distance from the observer to

the group increases, the projection of the group increases in size. No one ever received such an impression as this when viewing a series of similar objects in space.

209. — How, then, shall the statement made at the beginning of the chapter be reconciled with what at first seem to be existing proofs of its falsity? The explanation requires but a few words. Before any perspective projection can be made, the position of the observer's eye must be assumed. The whole construction is based upon this assumption, and the resulting perspective can be a true representation of the object in space only when it is viewed from the assumed position of the station point. In Figs. 86 and 87, the station points have purposely been taken so near the picture planes that there is very little chance of the observer placing his eye anywhere near the required positions, when looking at the perspectives, and the apparent distortion is exaggerated accordingly. Approximately correct views of these figures may be obtained by placing the eye very close to the paper and directly opposite the central object in each case.

210. — The necessary coincidence of the observer's eye with the assumed position of the station point, when viewing a perspective projection, has already been touched upon briefly on page 17 of the introduction, and in § 17, Chapter 1. The explanations there given assume the eye to be a simple point. As a matter of fact, the eye is a very complex instrument, and it is not the eye as a whole, but a definite point in the eye that must be made to coincide with the station point. The eye is really a miniature camera, fitted with a lens and with a sensitive surface called the retina, which receives the impression of the image.

211. — In Fig. 88, suppose the arrow (ab) to represent any object in space. Rays of light reflected from its surface enter the observer's eye, pass through some point o (the optic centre of the lens), and finally project an inverted image (a_1b_1) of the object on the retina as indicated. The retina, being sensitive to light, receives the impression of the image, which is conveyed by means of the optic nerve to the brain. It will be seen that this image is a conical projection of the object, the apex of the cone of visual projectors being at the point o. It is really a perspective projection of the object upon the retina, the station point coinciding with the point o, and the picture plane, instead of a plane, being the concave, approximately spherical surface of the retina. It is the point o which, strictly speaking, should coincide with the assumed position of the station point, when viewing a perspective drawing.

212. — The point *o* is between the retina and the object in space. This results, as it does in the camera, in the inversion of the perspective projection.[1]

213. — Now, suppose any plane (*PP*, Fig. 88) to be placed so as to intersect the cone of visual rays that project the object to the eye. This intersection will be the perspective of the object upon the plane *PP*. Suppose this perspective to be permanently fixed upon the plane. The object in space may be removed. The projection on the retina, instead of coming from the object, now comes from perspective projection on the plane *PP*. But as this perspective projection was formed by the visual rays which originally came from the object in space, the projection on the retina of the eye remains exactly as though the object were still in its original position, and the impression conveyed to the brain is exactly the same as though the original object were being viewed.[2]

214. — For convenience the perspective projection on the picture plane may be called the *intermediate perspective*, to distinguish it from the *final perspective* on the retina of the eye, from which the impression of the object is conveyed to the brain. Any perspective drawing should be considered as an intermediate perspective, the sole object of which is to cause the same final perspective on the retina of the eye that would be produced by the object in space.

Whenever the plane *PP* and the observer's eye occupy the same relative position that they did when the intermediate perspective was made (when the point *o* in the observer's eye is at the station point for the intermediate perspective), the observer will receive the same impression as though the original object were again in front of his eye, in its original position.

This gives a clear conception of what the perspective projection really is. It is simply a substitute for an object in space.

215. — It is evident that but one eye can be used when viewing a perspective projection from the correct position, for if one eye is in the correct position the view obtained with the other cannot be the same. If the right eye is being used to view an object, the left eye being farther to the left sees a slightly different view. More of the left-hand side of the object is visible, and less of the right, than is

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NOTE [1]. An image is always inverted on the retina of the eye, but the brain does not appreciate the inversion.

NOTE [2]. It is needless to state that correct color representation, atmospheric effect, brilliancy, distance, etc, are not considered in this discussion. The object in space is regarded in outline merely.

seen in the view obtained by the right eye. When an object in space is being viewed with both eyes, the projection received on the retina of one eye is slightly different from that on the other. This is always a fact, and in order absolutely to represent an object by perspective, and still use both eyes, two slightly different projections would have to be used, one for each eye. (See § 238.) It is evident, then, that with but one projection, but one eye at a time can occupy exactly the right position from which to view the projection. This is a fundamental rule of perspective. It must be obeyed, or the projection will not be a true representation of the object.

216. — It must not be forgotten that the perspective drawing itself is subject to all the laws of perspective, just as is any object that is being viewed. Consequently, as all parts of the perspective drawing are not the same distance from the observer, those parts which are farther from him have to be made relatively larger to allow for the increased foreshortening due to their increased distance from the station point.

For illustration, the perspectives of vertical lines are actually vertical lines, but the very fact that they are drawn parallel to one another on the picture plane insures that, as they recede from the station point, they will appear to the observer to converge. That is, the distance between the lines is made *relatively* larger than it is intended to appear, in proportion to their distance from the observer.

217. — For the same reason, the perspectives of all the circles in Fig. 89, the planes of which are parallel to the picture plane, are really of exactly the same size. These circles appear, when viewed from the station point, to diminish in area as the distance between them and the station point increases.

All this is taken care of in the perspective projection when it is viewed from the correct point, but when viewed from any other point the proper relations no longer seem to exist between the dimensions of the various parts of the projection, and the result is an apparent distortion.

218. — Fig. 90 shows the construction of a series of columns similar to the vertical cylinders in Fig. 87. This illustrates a phase of apparent distortion somewhat different from that explained in the

last two sections. It is caused by the varying angle between the picture plane and the base of the cone of visual rays which projects the column, this angle becoming more and more oblique as the column is situated farther and farther from the centre of the drawing. The station point (SP^V, SP^H) has been chosen very near the picture plane. It will be seen that the distances ab, which represent the widths of the perspectives of the different columns, increase rapidly as the columns they represent are situated farther and farther to the right and left of the centre. This is obviously caused by the change in the relation between the picture plane and the base of the cone of visual rays that projects the perspective of the column. It is evident, however, in every case, that the observer whose eye is at the station point, will see these varying widths at varying angles, each width being foreshortened according to its distance from his eye, in just such a proportion that they will all appear to cover exactly the columns which they represent. The moment his eye leaves the station point he appreciates more nearly the actual size of the projections, and they no longer represent to him a row of equal columns.

219. — When a row of columns in space is being viewed, the eye is turned directly towards each column as it comes under consideration. Thus the surface of the retina (or the picture plane of the eye) is brought into a position perpendicular to the visual ray from the axis of each column as it is considered. It is more nearly as though, instead of drawing the perspectives on the picture plane represented by HPP, the perspectives had been drawn upon a series of picture planes, represented in the figure by the lines HM, each of these planes being perpendicular to the horizontal projection of the visual ray through the centre of the column considered, and all being equally distant from the station point. The perspectives made on such a series of planes, when viewed from the station point, would cause exactly the same impression on the retina of the eye as would the perspectives constructed on PP. This is evident since both are projected by the same rays of light. The actual projections on the planes lettered HM, however, *as seen from some point other than the station point*, would more nearly agree with the general conception of what a row of columns should look like than would the perspectives on PP

220. — In the same way, when a sphere in space is viewed, the eye is turned directly towards it, and the visual ray from its centre is practically perpendicular to the retina of the eye. Thus a sphere in

space universally gives the impression of an object bounded by a circle. From § 184 it will be seen that the actual perspective projection of a sphere upon the picture plane will be a circle only when the visual ray through its centre is perpendicular to the picture plane. In other positions the perspective outline of a sphere is usually an ellipse. This is illustrated in Figs. 86 and 89. When the eye of the observer is at the assumed position of the station point, the long diameter of the ellipse will be seen at such an angle as to be foreshortened until it is just equal to the short diameter, and the ellipse will appear as a circle just coincident with the outline of the sphere in space that it represents. With the eye in any position other than the station point, the foreshortening of the major axis of the ellipse may be too great or too little, in either of which cases the ellipse will not appear as a circle, and, consequently, will not give the observer the impression of a sphere.

221. — For the same reason that a sphere in space always appears circular in form, a horizontal circle always appears as a horizontal ellipse, *i.e.*, an ellipse the major axis of which is horizontal. In the perspective projection on the picture plane, however, this is by no means true. A horizontal circle is projected upon the picture plane as a horizontal ellipse only when the horizontal projection of a visual ray through its centre is perpendicular to *HPP*. If the circle is to the right or left of the station point, the axes of the ellipse which represents its perspective, will be more or less inclined. This effect is seen in Fig. 87 in the horizontal bases of the circular plinths on which the cylinders are resting. The centre circle is the only one projected as a horizontal ellipse. The inclination of the axes of the ellipses representing the perspectives of some of the spheres in Fig. 86 is also very apparent. Yet these are true perspectives, and would convey the correct impression if the eye could be placed exactly at the station point.[1]

222. — All parts of the projection are apparently more or less distorted when viewed from a point other than the station point. The distortion in plane surfaces, however, is generally not so noticeable,

NOTE [1]. In a series of equal circles, the planes of which are all parallel to the picture plane and equally distant from it, it is evident that the intersection of the picture plane with the cone of visual rays that projects the circle will, in each case, be parallel to the base of the cone. The perspectives of such a series of circles will not, therefore, be affected by the phase of apparent distortion described in § 221, but all will be exactly similar in shape (§ 179-*a*) and size. The only apparent distortion to which they will be subjected being similar to that described in § 216. This is illustrated in Fig. 89 by the perspectives of the bases of the circular plinths which are parallel to the picture plane.

or if noticed is not so disagreeable, as in regular curved forms, such as the circle or the sphere.

223. — The student must now see that the so-called distortion in a perspective drawing is only an apparent condition, and really caused by a misconception on the part of the observer as to what a perspective really is. This is a misconception which is very general, however, and one that must be met. It is extremely unlikely that every one viewing a perspective should understand its limitations, and even if this were not so, it is often impossible to determine from simple inspection the assumed position of the station point. It is most improbable that the casual observer will put one eye in the correct position, and shut the other eye, while viewing a drawing. Thus the subject of apparent distortion, although *in no way affecting the real accuracy of a perspective projection*, becomes a most important subject for consideration. Every care should be taken to prevent such unpleasant effects as those shown in Figs. 86 and 87.

224. — In the first place the apparent distortion is greatly affected by the choice of the position of the station point before beginning the drawing. The aim should be to choose this so that the observer will most naturally place his eye, or rather his eyes, nearly at the correct point. This is not so difficult a task as it at first appears. It is always natural to hold a drawing with its centre directly in front of us while looking at it. Therefore, the first rule to be observed is to choose the station point somewhere directly in front of the middle of the perspective projection.

225. — In the normal eye the distance of distinct vision is about ten inches; that is, an observer, when looking at a drawing, will naturally place it about ten inches in front of his eyes. This applies only when the drawing is small, however. As the drawing increases in size the observer naturally holds it farther and farther from him, in order to embrace the whole without having to turn his eye too far to the right or left. Sometimes a general rule is given to make the distance of the station point equal to the altitude of an equilateral triangle, having the extreme dimensions of the drawing for its base and the station point for its apex. This rule is purely arbitrary, but is, perhaps, as good a general guide as any. It is very seldom that

an observer will naturally place his eye nearer to the drawing than the distance of distinct vision; therefore, it is a fair general rule to make the minimum distance between the picture plane and station point ten inches.

226. — The apparent distortion is always greater when the observer's eye is too far away from the perspective projection than when it is too near. In the former case the objects do not seem to diminish sufficiently in size as they recede from the eye; in some instances, for example, Fig. 87, the objects actually appear to increase in size as they are taken farther and farther from the eye. On the other hand, when the eye is between the station point and the picture plane, the effect is to make the objects diminish in size somewhat too rapidly as they recede from the eye. This effect is not so easily appreciated, and if appreciated it is not nearly so disagreeable as that produced by viewing the perspective from a point too far from the picture plane. Therefore, it is better to assume the station point too far from the picture plane than too near.

227. — Having decided upon the position of the station point, the view to be presented in the perspective projection should receive consideration. It has been seen that the apparent distortion is most disagreeable in regular curved forms, and that it is more noticeable near the edges than at the centre of a drawing. For these reasons, care should be taken to place all curved forms as near the centre of the drawing as possible, thus reducing their apparent distortion to a minimum.

228. — Finally, it is customary to introduce certain so-called corrections as an aid in minimizing the disagreeable effects produced by viewing the drawing from the wrong point. These are in reality no corrections at all. They are absolute transgressions of the rules of perspective, in order to make the drawing appear approximately correct when viewed from a wrong position. A perspective projection so altered will not be absolutely correct from any point of view; but, on the other hand, it may not be disagreeable from any point of view that is likely to be taken by an observer.

229. — These alterations usually consist in making the perspectives of all horizontal circles horizontal ellipses, or at least so nearly horizontal that the inclination of their principal axes will not intrude itself upon the notice of the observer. The perspective of a sphere is

always made a circle, the laws of perspective being used only to establish its position and its dimensions.

230. — If figures of men or animals occur, their position and approximate dimensions are established by the rules of perspective, and the figures then drawn in by eye. Other parts of the drawing, as, for instance, the diameters of the columns in Fig. 87, may be altered just sufficiently to prevent their inequalities from being noticed by the observer.

231. — Great care must be used in altering a perspective drawing, and the distortion should first be reduced to a minimum by proper assumptions in regard to the position of the station point and to the view to be shown in the perspective projection. It will readily be seen that any change made in one part of a drawing will affect its relation to every other part. Thus, in Fig. 87, if the outer cylinders were cut down to the same diameter as the centre one, they would seem too small in relation to the surrounding parts of the drawing, where a distortion somewhat similar to that in the cylinders really exists but is not so apparent. A new drawing should be laid out with the station point assumed much farther from the picture plane.[1]

The diameters of the outer cylinders may then be slightly diminished; the diameter of the centre cylinder may be increased slightly until an approximately equal relation is established. The ellipses which form the bases of the plinths cannot, perhaps, be drawn with their major axes perfectly horizontal without creating new distortions just as disagreeable as those we are endeavoring to correct. They should be drawn with their major axes approaching the horizontal just enough to cheat the observer into believing that they are so. It will be seen that by diminishing the diameters of the outer columns more of the background will be made to appear than would actually be visible. This, however, will seldom give much trouble, unless the apparent distortion is much exaggerated.

232. — Following out the idea expressed in §§ 218 and 219, it has been proposed to use a concave, cylindrical surface as a substitute for the ordinary picture plane. If the surface used is that of a right,

NOTE [1]. The result of so doing may be seen by comparing Figs. 91 and 92. Fig. 91 shows a perspective made with the station point at a distance from the picture plane, but slightly greater than the length of one of the cylinders. In Fig. 92 the station point has been taken much farther from the picture plane, with a consequent decrease in the apparent distortion.

circular cylinder with a vertical axis, and if the station point is chosen on this axis, it is evident that the horizontal projection of the visual ray to any point of the surface will be normal to the surface.

In the perspective projection on such a surface, the phases of distortion described in §§ 216, 217 and 218, so far as horizontal extension is concerned, will practically disappear. This is illustrated in Fig. 93. At the upper part of this figure, a series of vertical columns is shown in plan. The position of the station point is indicated by SP^H. The perspectives of the columns on the right of the line *MN* have been projected upon the plane surface represented by *HPP*, while the perspectives of those on the left of *MN* have been projected upon the right, circular cylindrical surface represented by *HCS*. The width of the perspective projection of each column is indicated by the distance (*ab*) intercepted on *HPP* or on *HCS* by the tangent visual rays.

It will be seen that the widths of the perspectives on the plane surface *increase* as the distance between them and the station point *increases*. This is exactly *opposite* to the impression received by an observer when viewing such a series of columns in space. This has already been explained in § 218.

On the cylindrical surface, however, the widths of the perspectives will be seen to *decrease* as the distance between them and the station point *increases*. This is exactly *in accord* with the impression received by an observer when viewing such a series of columns in space. Thus, so far as horizontal extension merely is concerned, the cylindrical surface of projection shows some advantages over the plane surface.

233.—In regard to vertical extension, however, this is not true. As the elements of the cylindrical surface are vertical lines, it is evident that in a vertical direction the phenomena described in §§ 201 to 207 inclusive will apply equally well to the cylindrical surface of projection as to the plane surface usually employed.[1]

234.—In addition to the apparent distortions that are noticed in a perspective projection on a plane surface, the perspective on a cylindrical surface, when viewed from any point other than the station point, presents a new kind of apparent distortion even more disagreeable perhaps than any yet described.

NOTE [1]. A concave spherical surface of projection might be employed, with the station point chosen at the centre of the sphere. Such a surface would, at every point, be normal to the visual ray at that point. The practical disadvantages of using such a surface of projection would obviously outweigh any advantages that might be gained, and, after all, the eye must be exactly at the centre of the sphere in order to receive a correct impression from the perspective projection upon its surface.

As the perspective of any straight line is formed by the intersection of a plane with the surface of projection, it follows that the perspective of every straight line (except a vertical one) upon a vertical cylindrical surface will be an ellipse. The curvature of such an ellipse will be noticeable except when viewed from the assumed position of the station point. With the eye in any other position, the perspectives of all straight lines will appear curved. This has given rise to the term Curvilinear Perspective, which is often applied to a perspective projection upon a cylindrical surface.

235. — Furthermore, in order to make use of the cylindrical perspective as conveniently as though it were on a plane, the cylinder, after receiving the projection, must be developed or flattened out to coincide with the plane of the paper. This operation evidently makes it impossible to find any one point from which to get a correct view of the perspective.

In its original cylindrical form the perspective would appear absolutely correct to the observer whose eye was at the station point. Viewed from this point, all the curves which represent straight lines would seem to be straight, since the planes of these curves would all pass through the observer's eye. But, after development, the only straight lines which will appear straight in the perspective projection are vertical lines and lines lying in the plane of the horizon.

236. — Fig. 94 shows the developed perspective projection on a cylinder. All straight lines, except the vertical ones and those on a level with the eye, are curved. This curvature may, evidently, be lessened by taking the station point farther from the projecting surface; or, in other words, increasing the radius of the cylinder that forms the projecting surface.

If the view shown in Fig. 94 could be bent back into the cylindrical form, and the eye placed on the axis of the cylinder at the original position of the station point, the perspective projection would appear absolutely correct.

237. — The cylindrical picture plane has a legitimate use in the so-called "Cyclorama." The painted panorama, which is to be viewed, is hung on the wall of a cylindrical room. It is really an immense perspective, made with the cylindrical wall for a surface of projection. The observer is confined to the approximately correct position by a small raised platform built on the axis of the room, and at such a height that his eye is brought to a level with the assumed horizon. The space between the painted background and the plat-

form occupied by the observer, is usually filled with actual objects completing the foreground of the panorama. When well carried out the illusion is very perfect, and it is difficult for the observer on the platform to determine where the real objects leave off and the perspective representations begin.

238. — A hint of the principle of Binocular Perspective was given in § 215. In viewing an object in space, an observer uses both eyes, and on the retina of each eye receives an impression which differs from that on the other. It is a combination of these two slightly different impressions that gives the idea of relief or solidity to the object viewed. If both eyes are used in viewing a single projection on a plane surface, one eye can see no more or less than the other. If, on the other hand, both eyes are used to view some object in space, the difference between the two views obtained on the retinas of the eyes gives the assurance of solidity. The difference between the two views is the same that would be obtained if two perspective projections of one object were made from two station points assumed from two and one-half to three inches apart. Therefore, if two perspectives of an object are drawn, one representing the view as seen by the left eye, the other representing the view as seen by the right, and then if these two slightly dissimilar views are placed before the eyes in such a manner that each view is presented to the eye for which it was made, the impression received by the brain should be that of the actual object in space, seen in relief. That such indeed is a fact may be proved by experiment.

In Fig. 95 are shown two perspectives. Each was made from the same group of objects, but the station points for the two were some three inches apart. The slight differences between the views can easily be detected. If the student will hold a card between the two views, perpendicular to the plane of the paper, in such a manner that the right eye cannot see the left-hand view, nor the left eye see the right-hand view, and if he will then gaze intently for a few seconds at the corresponding points (marked by a black dot in each view), the two projections will seem to merge into a single one, having the unmistakable appearance of relief.

This is the most perfect form of perspective projection. The principle here illustrated is used in the instrument called the Stereoscope, which furnishes a rather more elaborate method of obtaining the result just described.

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